格子 QCD による S=−2 バリオン間相互作用のクォーク質量依存性の研究

Kenji Sasaki (*CCS, University of Tsukuba*)

for HAL QCD collaboration

HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration

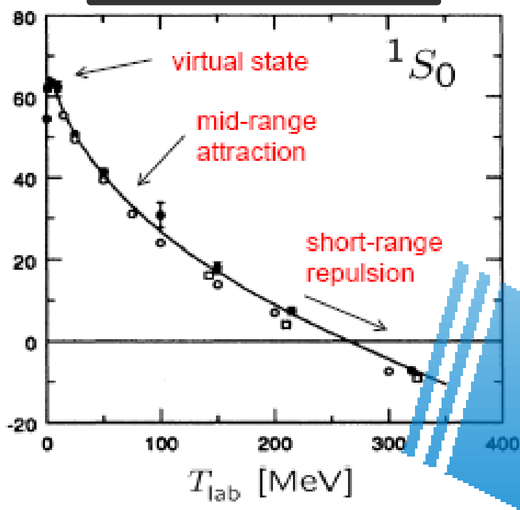
S. Aoki (*Univ. of Tsukuba*)
B. Charron (*Univ. of Tokyo*)
T. Doi (*RIKEN*)
F. Etminan (*Univ. of Tsukuba*)
T. Hatsuda (*RIKEN*)
Y. Ikeda (*Tokyo Inst. Tech.*)
T. Inoue (*Nihon Univ.*)
N. Ishii (*Univ. of Tsukuba*)
K. Murano (*RIKEN*)
H. Nemura (*Univ. of Tsukuba*)
M. Yamada (*Univ. of Tsukuba*)

Introduction

Introduction

Baryon-baryon interactions are crucial for nuclear and astrophysics

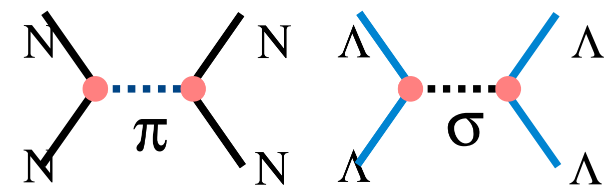
BB phase shift



For NN, large amount of scattering data
For YN and YY, experimental data are scarce.

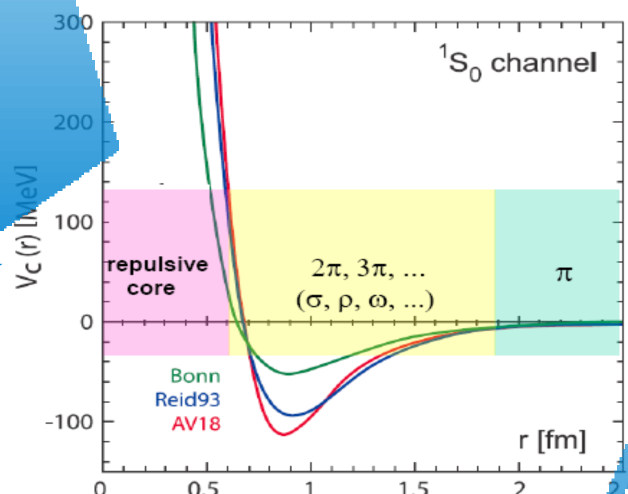
Meson exchange model

Described by hadron dof
with phenomenological repul. core



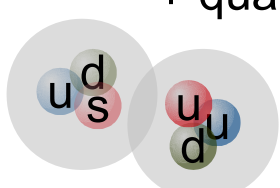
Repulsive core is generated
by ω meson exchange.

BB interaction (potential)



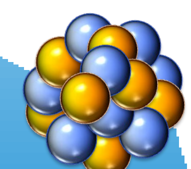
Quark cluster model

Effective meson ex
+ quark anti-symmetrization

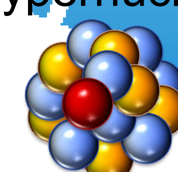


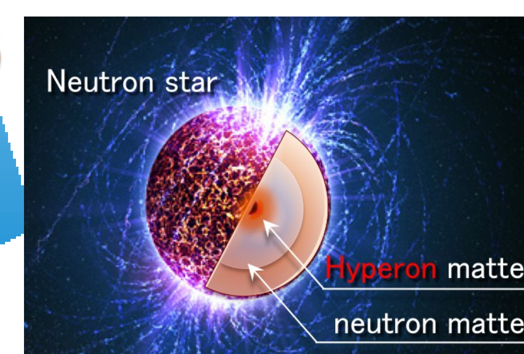
Quark Pauli effects
are taking into account

Nucleus



Hypernucleus



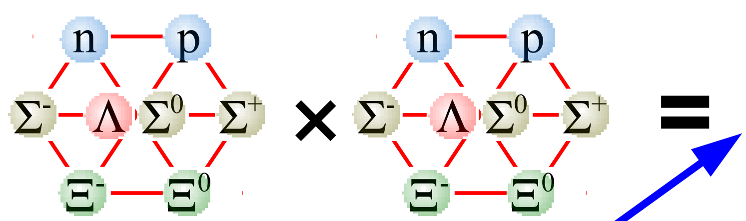


Introduction

Study the hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions

Strangeness brought the deeper understanding of BB interaction.

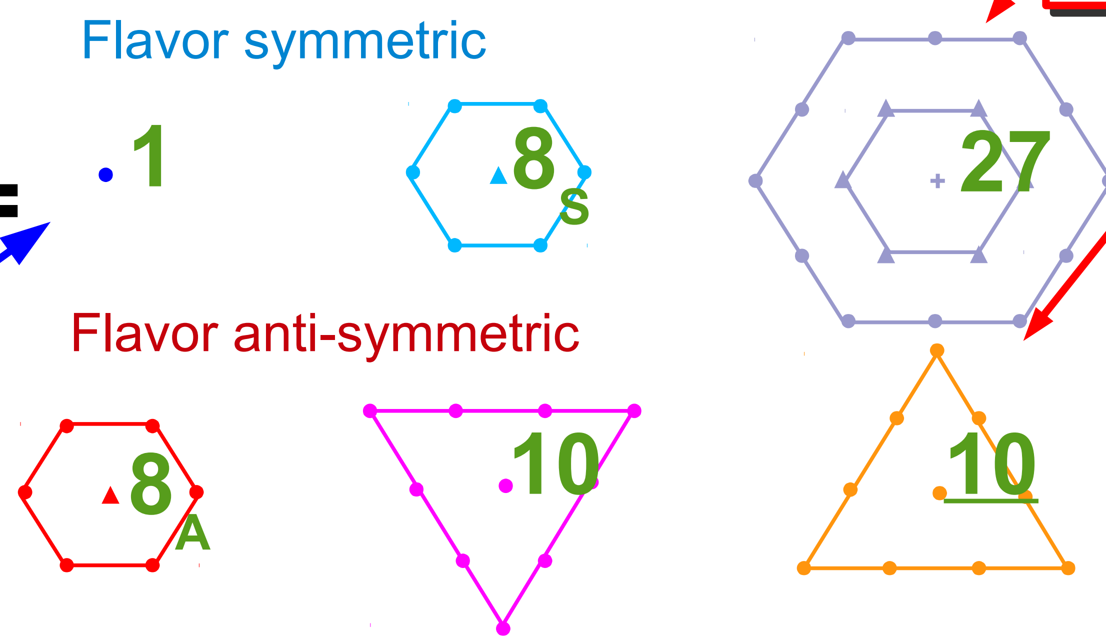
Three flavor (u,d,s) world



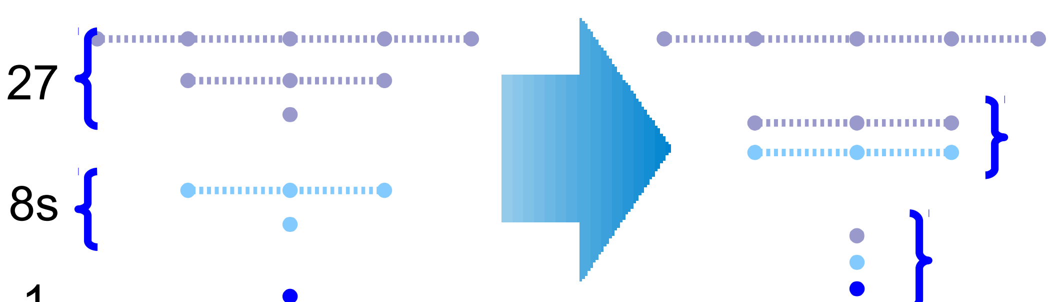
H-dibaryon state is expected

Wide variety of BB interaction

NN sector



SU(3) breaking



I=1 states
8s, 27 mixing

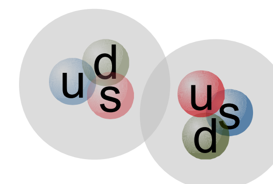
I=0 states
1, 8s, 27 mixing

Introduction

S=-2 BB interaction

- Direct scattering experiment is very difficult.
- All irreducible representations are involved.

H-dibaryon



- Flavor singlet object (6q / tightly bound)
  - No Pauli blocking was proved by quark cluster model.
  - Strong color magnetic (attractive) interaction

Recent Lattice QCD studies

- HAL QCD: SU(3) limit

  BE = 26MeV m_π = 470MeV

- NPLQCD: SU(3) breaking

  BE = 13MeV m_π = 390MeV

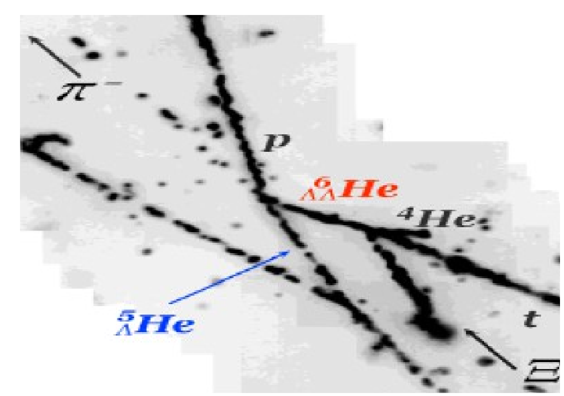
Conclusions of the "NAGARA Event"

K.Nakazawa and KEK-E176 & E373 collaborators

Λ–N attraction

Λ–Λ weak attraction

$m_H \geq 2m_\Lambda$ - 6.9MeV



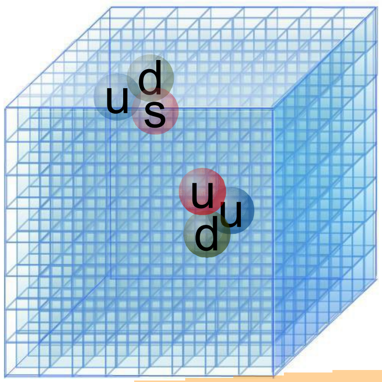
HAL QCD strategy

QCD to hadronic interactions

QCD Lagrangian

$$L_{QCD} = \bar{q}\left(i\gamma_\mu D^\mu - m\right)q + \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$$

Lattice QCD simulation



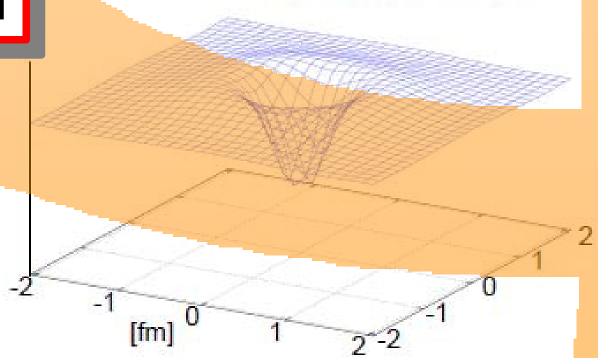
$$\langle O(\bar{q}, q, U)\rangle = \int e^{-S_E}\, O\, dU\, d\bar{q}\, dq$$

Lattice QCD simulation can connect the fundamental QCD with nuclear physics

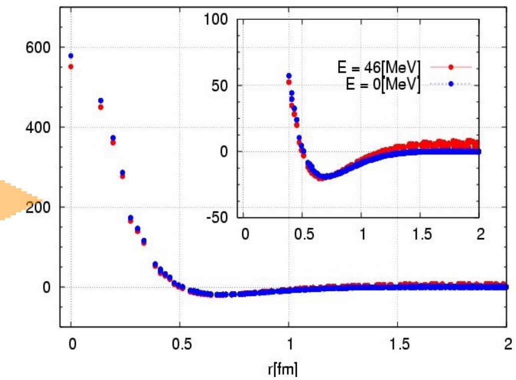
The potential through our method reproduce to the phase shift by QCD

HAL QCD method

NBS wave function



BB interaction (potential)



N. Ishii, S. Aoki and T. Hatsuda, Phys. Rev. Lett. **99** (2007) 022001

Nambu-Bethe-Salpeter wave function

Definition : equal time NBS w.f.

$$\Psi_\nu(E, t-t_0, \vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(t, \vec{x}+\vec{r}) B_j(t, \vec{x}) | E, \nu, t_0 \rangle$$

$$B = \epsilon^{abc} (q_a^T C \gamma_5 q_b) q_c$$

The ket stands for the eigenstate of the complete set of observables

E : Total energy of system
ν : other observables which needs to form the complete set

Local composite interpolating operators

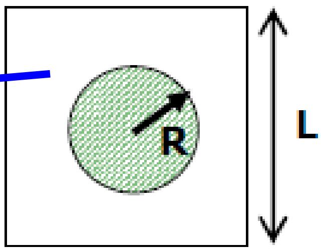
$$p_\alpha = \epsilon_{c_1 c_2 c_3} (C\gamma_5)_{d_1 d_2} \delta_{d_3 \alpha} u(\xi_1) d(\xi_2) u(\xi_3)$$

$$\Sigma^0_\alpha = -\epsilon_{c_1 c_2 c_3} (C\gamma_5)_{d_1 d_2} \delta_{d_3 \alpha} \sqrt{\frac{1}{2}} [d(\xi_1) s(\xi_2) u(\xi_3) + u(\xi_1) s(\xi_2) d(\xi_3)]$$

$$\Lambda_\alpha = -\epsilon_{c_1 c_2 c_3} (C\gamma_5)_{d_1 d_2} \delta_{d_3 \alpha} \sqrt{\frac{1}{6}} [d(\xi_1) s(\xi_2) u(\xi_3) + s(\xi_1) u(\xi_2) d(\xi_3) - 2u(\xi_1) d(\xi_2) s(\xi_3)]$$

NBS wave function has a same asymptotic form with quantum mechanics.
(NBS wave function is characterized from phase shift)

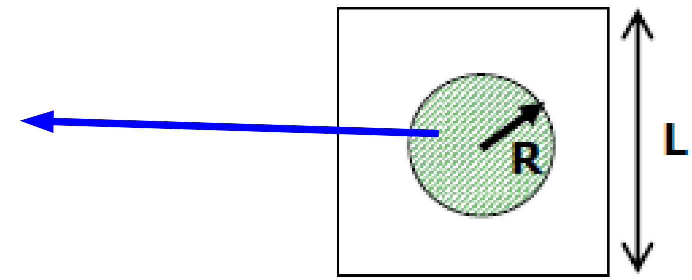
$$\Psi(t-t_{0,} \vec{r}) \simeq A \frac{\sin(pr + \delta(E))}{pr}$$



Schrödinger equation

- Define the energy-independent potential in Schrödinger equation (most general form)

$$\left(\frac{k^2}{2\mu} - H_0\right)\Psi(\vec{x}) = \int U(\vec{x},\vec{y})\Psi(\vec{y})\,d^3y$$



- Recent development : Time dependent method.

We replace ψ to *R* defined below

$$\partial_t R_\alpha(\vec{x},E) \equiv \partial_t\left(\frac{A\Psi_\alpha(\vec{x},E)e^{-Et}}{e^{-m_A t}e^{-m_B t}}\right) \propto -\frac{p_\alpha^2}{2\mu_\alpha}R_\alpha(\vec{x},E)$$

- Performing the derivative expansion for the interaction kernel

$$\left(-\frac{\partial}{\partial t} - H_0\right)R(\vec{x}) = \int U(\vec{x},\vec{y})R(\vec{y})\,d^3y$$

- Taking the leading order of derivative expansion of non-local potential

$$U(\vec{x},\vec{y}) \simeq V_0(\vec{x})\delta(\vec{x}-\vec{y}) + V_1(\vec{x},\nabla)\delta(\vec{x}-\vec{y})\ \cdots$$

- Finally local potential was obtained as

$$V(\vec{x}) = -\frac{\partial_t R(\vec{r})}{R(\vec{v})} + \frac{1}{2\mu}\frac{\nabla^2 R(\vec{x})}{R(\vec{x})}$$

Coupled channel Schrödinger equation

Preparation for the NBS wave function

Two-channel coupling case

$$\Psi^{\alpha}(E,t,\vec{r}) = \sum_{\vec{x}} \langle 0 | (B_1 B_2)^{\alpha}(t,\vec{r}) | E \rangle$$

$$\Psi^{\beta}(E,t,\vec{r}) = \sum_{\vec{x}} \langle 0 | (B_1 B_2)^{\beta}(t,\vec{x}) | E \rangle$$

The same "in" state

Inside the interaction range

In the *leading order of velocity expansion* of non-local potential,

Coupled channel Schrödinger equation.

Factorization of interaction kernel

$$\left(\frac{p_{\alpha}^2}{2\mu_{\alpha}} + \frac{\nabla^2}{2\mu_{\alpha}}\right)\psi^{\alpha}(\vec{x},E) = V^{\alpha}{}_{\alpha}(\vec{x})\psi^{\alpha}(\vec{x},E) + V^{\alpha}{}_{\beta}(\vec{x})\psi^{\beta}(\vec{x},E)$$

μ_{α} : *reduced mass*

p_{α} : asymptotic momentum.

Asymptotic momentum are replaced by the time-derivative of *R*.

$$R_I^{B_1 B_2}(t,\vec{r}) = \sum_{\vec{x}} \langle 0 | B_1(t,\vec{x}+\vec{r}) B_2(t,\vec{x}) \bar{I}(0) | 0 \rangle e^{(m_1+m_2)t}$$

$$\begin{pmatrix} V^{\alpha}{}_{\alpha}(\vec{r}) & V^{\alpha}{}_{\beta}(\vec{r})x \\ V^{\beta}{}_{\alpha}(\vec{r})x^{-1} & V^{\beta}{}_{\beta}(\vec{r}) \end{pmatrix} = \begin{pmatrix} \left(\frac{\nabla^2}{2\mu_{\alpha}} - \frac{\partial}{\partial t}\right) R_{I1}^{\alpha}(\vec{r},E) & \left(\frac{\nabla^2}{2\mu_{\beta}} - \frac{\partial}{\partial t}\right) R_{I2}^{\beta}(\vec{r},E) \\ \left(\frac{\nabla^2}{2\mu_{\alpha}} - \frac{\partial}{\partial t}\right) R_{I1}^{\alpha}(\vec{r},E) & \left(\frac{\nabla^2}{2\mu_{\beta}} - \frac{\partial}{\partial t}\right) R_{I2}^{\beta}(\vec{r},E) \end{pmatrix} \begin{pmatrix} R_{I1}^{\alpha}(\vec{r},E) & R_{I1}^{\beta}(\vec{r},E) \\ R_{I2}^{\alpha}(\vec{r},E) & R_{I2}^{\beta}(\vec{r},E) \end{pmatrix}^{-1}$$

$$x = \frac{\exp(-(m_{\alpha_1}+m_{\alpha_2})t)}{\exp(-(m_{\beta_1}+m_{\beta_2})t)}$$

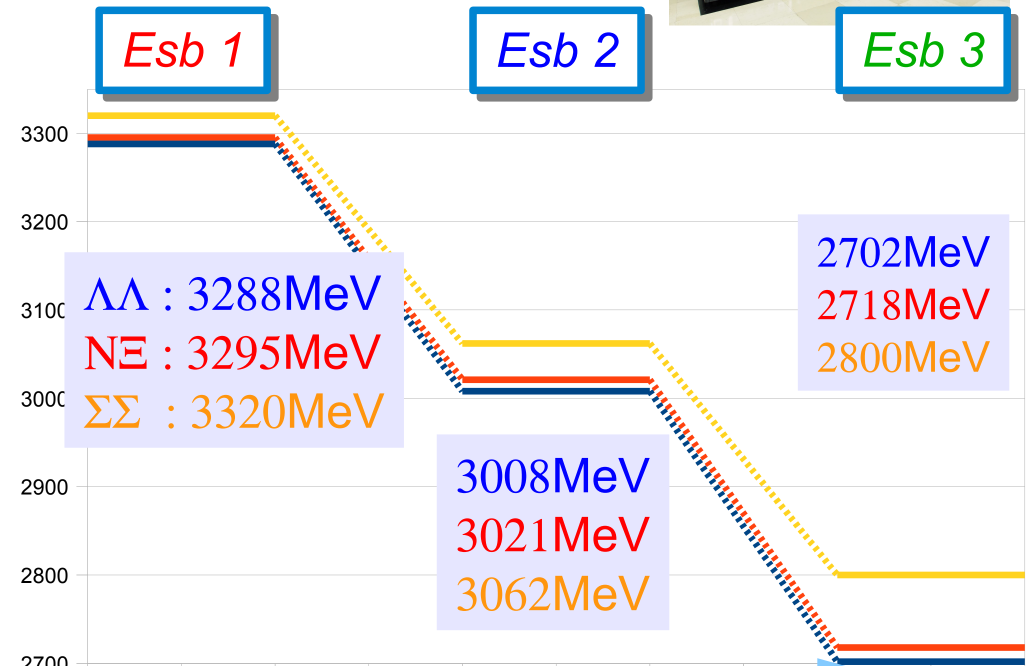
Results

Numerical setup

- $2+1$ flavor gauge configurations by PACS-CS collaboration.
  - RG improved gauge action & O(a) improved clover quark action
  - $\beta = 1.90$, $a^{-1} = 2.176$ [*GeV*], 32^3x64 lattice, $L = 2.902$ [*fm*].
  - $\kappa_s = 0.13640$ is fixed, $\kappa_{ud} = 0.13700$, 0.13727 and 0.13754 are chosen.
- Flat wall source is considered to produce S-wave B-B state.
- The KEK computer system A resources are used.

In unit of MeV	*Esb 1*	*Esb 2*	*Esb 3*
π	701±1	570±2	411±2
K	789±1	713±2	635±2
m_π/m_K	0.89	0.80	0.65
N	1585±5	1411±12	1215±12
Λ	1644±5	1504±10	1351± 8
Σ	1660±4	1531±11	1400±10
Ξ	1710±5	1610± 9	1503± 7

u,d quark masses lighter

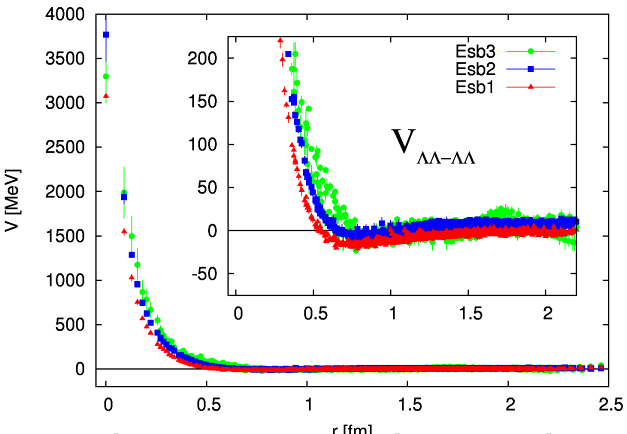


SU(3) breaking effects becomes larger

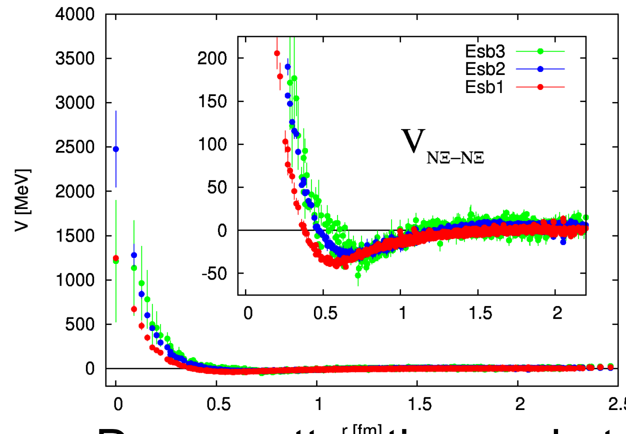
ΛΛ, NΞ, ΣΣ (I=0) $^1S_0$ channel

Esb1 : mπ= 701 MeV
Esb2 : mπ= 570 MeV
Esb3 : mπ= 411 MeV

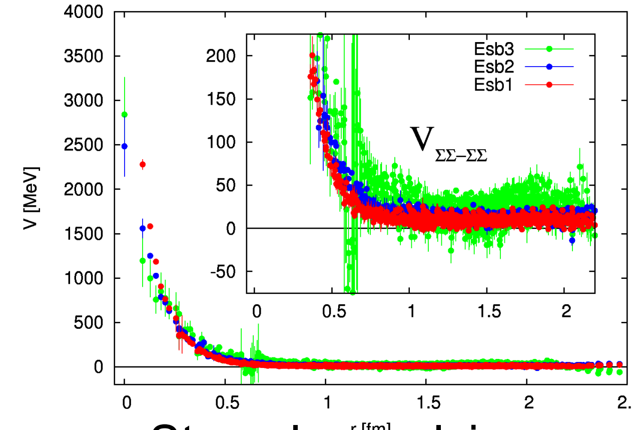
Diagonal elements



shallow attractive pocket



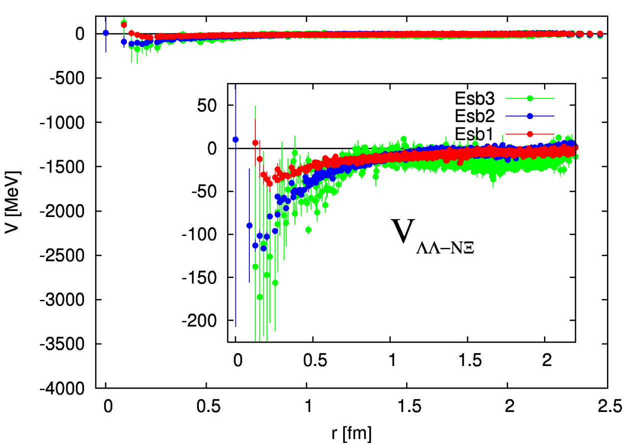
Deeper attractive pocket

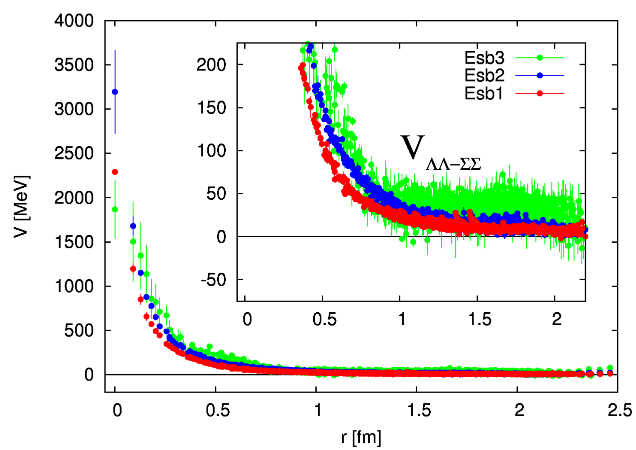


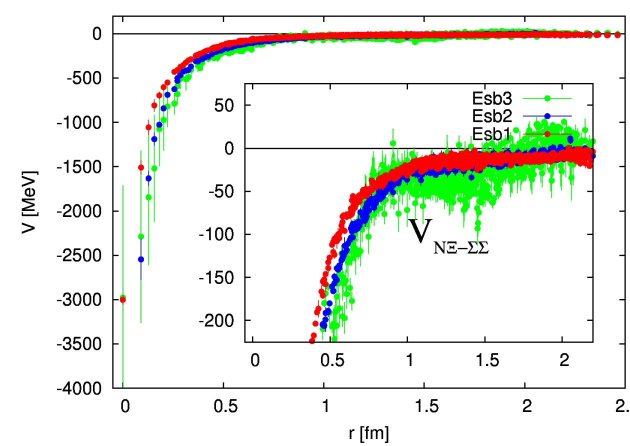
Strongly repulsive

All channels have repulsive core

Off-diagonal elements







Relatively weaker than the others

In this channel, our group found the "H-dibaryon" in the SU(3) limit.

Comparison of potential matrices

Transformation of potentials
from the particle basis to the SU(3) irreducible representation (IR) basis.

SU(3) Clebsh-Gordan coefficients

$$\begin{pmatrix}|1\rangle\\|8\rangle\\|27\rangle\end{pmatrix} = U\begin{pmatrix}|\Lambda\Lambda\rangle\\|N\Xi\rangle\\|\Sigma\Sigma\rangle\end{pmatrix}, \quad U\begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{N\Xi} & V^{\Lambda\Lambda}_{\Sigma\Sigma}\\ V^{N\Xi}_{\Lambda\Lambda} & V^{N\Xi} & V^{N\Xi}_{\Sigma\Sigma}\\ V^{\Sigma\Sigma}_{\Lambda\Lambda} & V^{\Sigma\Sigma}_{N\Xi} & V^{\Sigma\Sigma}\end{pmatrix} U^t \to \begin{pmatrix} V_1 & & \\ & V_8 & \\ & & V_{27}\end{pmatrix}$$

In the SU(3) irreducible representation basis,
the potential matrix should be diagonal in the SU(3) symmetric configuration.

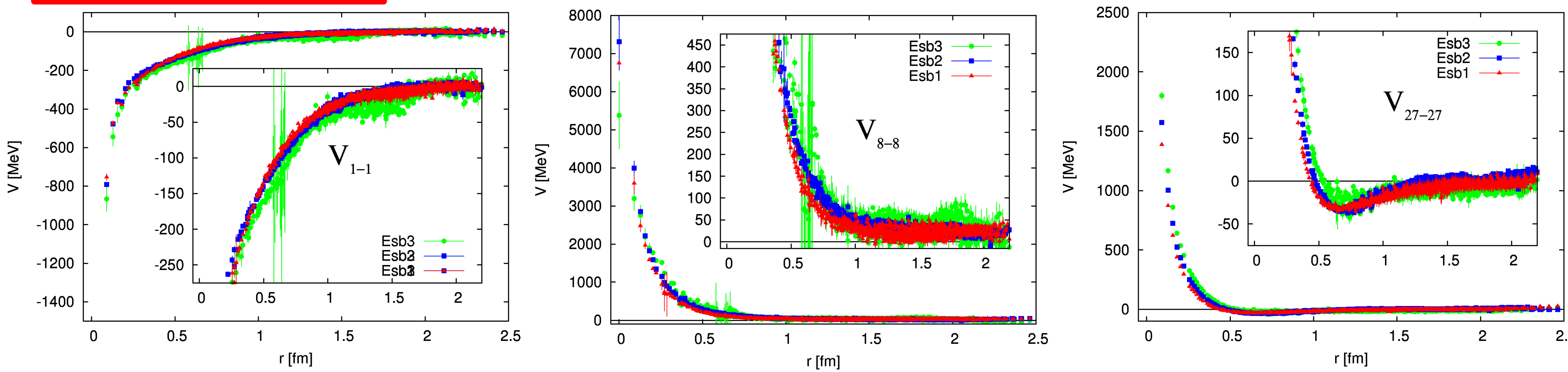
Off-diagonal part of the potential matrix in the SU(3) IR basis
would be an effectual measure of the SU(3) breaking effect.

We will see how the SU(3) symmetry of potential will be broken
by changing the u,d quark masses lighter.

1, 8_s, 27 (I=0) $^1S_0$ channel

Esb1 : mπ= 701 MeV
Esb2 : mπ= 570 MeV
Esb3 : mπ= 411 MeV

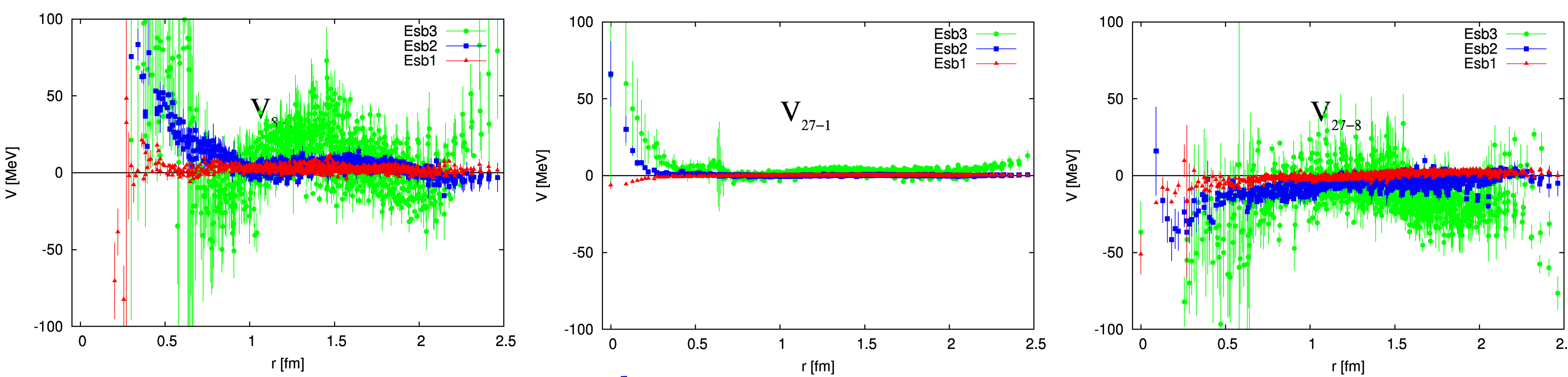
Diagonal elements



Strongly attractive
H-dibaryon channel

Pauli blocking effect

Off-diagonal elements

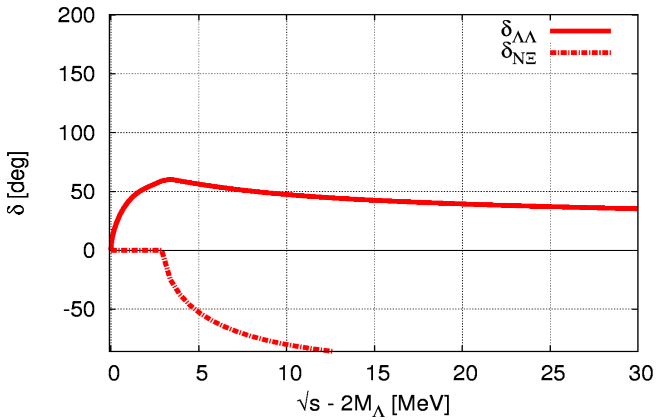


Mixture of singlet and octet
Is relatively larger than the others

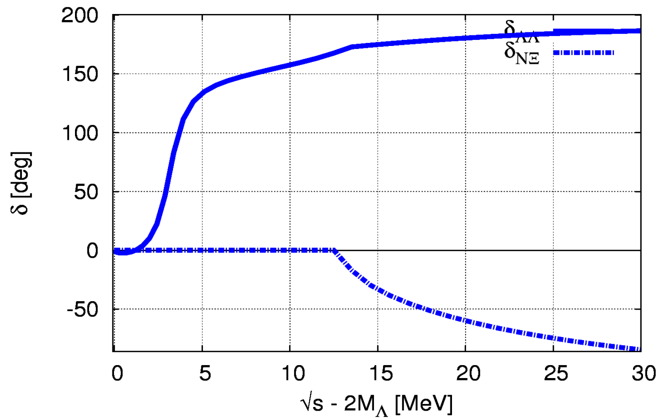
27 plet does not mix so much to the other representations

$\Lambda\Lambda$ and $N\Xi$ phase shifts

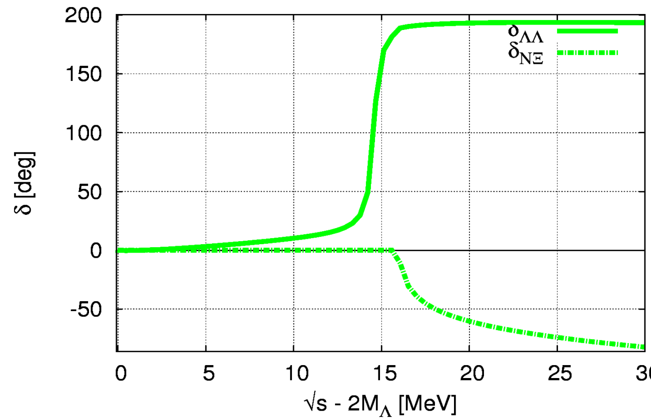
Esb1 : m_π= 701 MeV



Esb2 : m_π= 570 MeV



Esb3 : m_π= 411 MeV



Preliminary!

- Esb1:
  - Bound H-dibaryon
- Esb2:
  - H-dibaryon is near the $\Lambda\Lambda$ threshold
- Esb3:
  - The H-dibaryon resonance energy is close to $N\Xi$ threshold..

- We can see the clear resonance shape in $\Lambda\Lambda$ phase shifts for Esb2 and 3.
- The "binding energy" of H-dibaryon from $N\Xi$ threshold becomes smaller as decreasing of quark masses.

Summary and outlook

- We have investigated the S=-2 BB system from lattice QCD.

- In order to deal with a variety of interactions, we extend our method to the coupled channel formalism.

- Potentials are derived from NBS wave functions calculated with PACS-CS configurations

- Quark mass dependence of potentials can be seen not in long range region but in shot distances as an enhancement of repulsive core.

- Small mixture between different SU(3) IRs can be seen as the flavor SU(3) breaking effect.

- SU(3) breaking effects are still small even in $m\pi/mK=0.65$ situation but it would be change drastically at physical situation $m\pi/mK=0.28$.