# 格子シミュレーションを用いた QCDを超えるゲージ理論に対する最近の研究

arXiv:1212.1353[hep-lat] and work in progress

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2013/03/07@ HPCI戦略分野５全体シンポジウム(秋葉原)

Numerical simulation was carried out
on

NEC SX-8 and SR16000 @ YITP, Kyoto
NEC SX-8 @ RCNP, Osaka
SR11000 and BlueGene/L
SR16000 and BlueGene/Q @ KEK
100 GPUs @ Osaka and Taiwan

also use JLDG

# Introduction

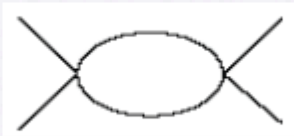
Higgs sector in the Standard Model Lagrangian

$$\mathcal{L}_H \sim \frac{1}{2} D_\mu \phi D^\mu \phi^\dagger + \frac{\lambda}{4}(\phi\phi^\dagger - v^2)^2$$

Problem with a fundamental Higgs boson

Hierarchy problem (need fine-tuning to cancel a quadratic divergence)

Triviality problem



$\Rightarrow$ $\beta(\lambda) = \frac{3\lambda^2}{2\pi^2} > 0$

No interaction at low energy

Running coupling constant diverges at a finite energy

Cuttoff theory?

Candidates for the origin of Higgs sector

Supersymmetry

Extra dimension

Walking techni-color

Fourth generation

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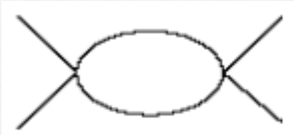

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Google
"Extra dimension 2013"

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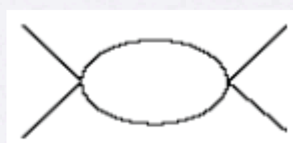

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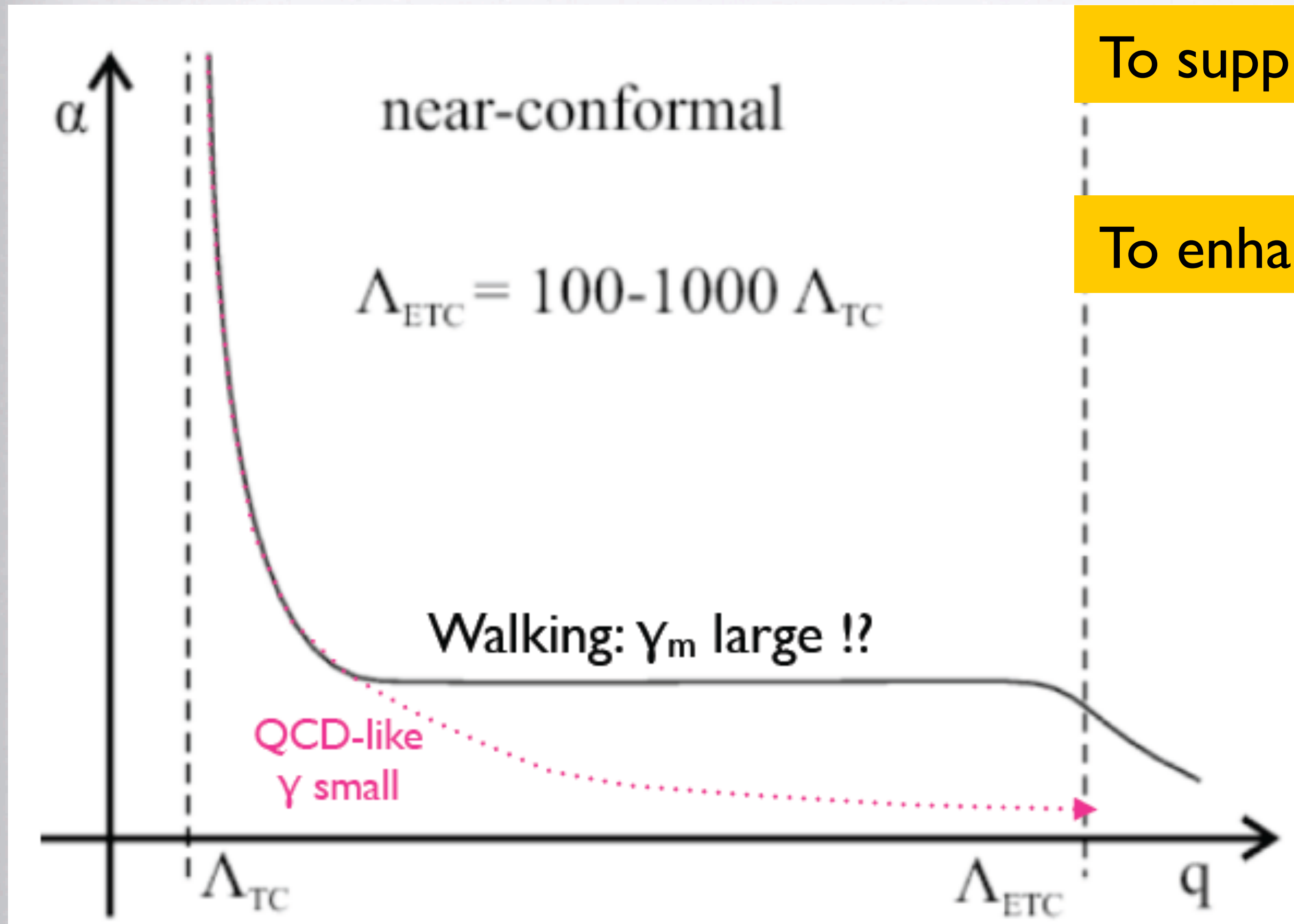
Introduce an additional gauge interaction and fermions

# Walking Technicolor




To suppress FCNC process

To enhance quark mass

Asymptotic free (UV complete)

IR fixed point (or walking behavior)

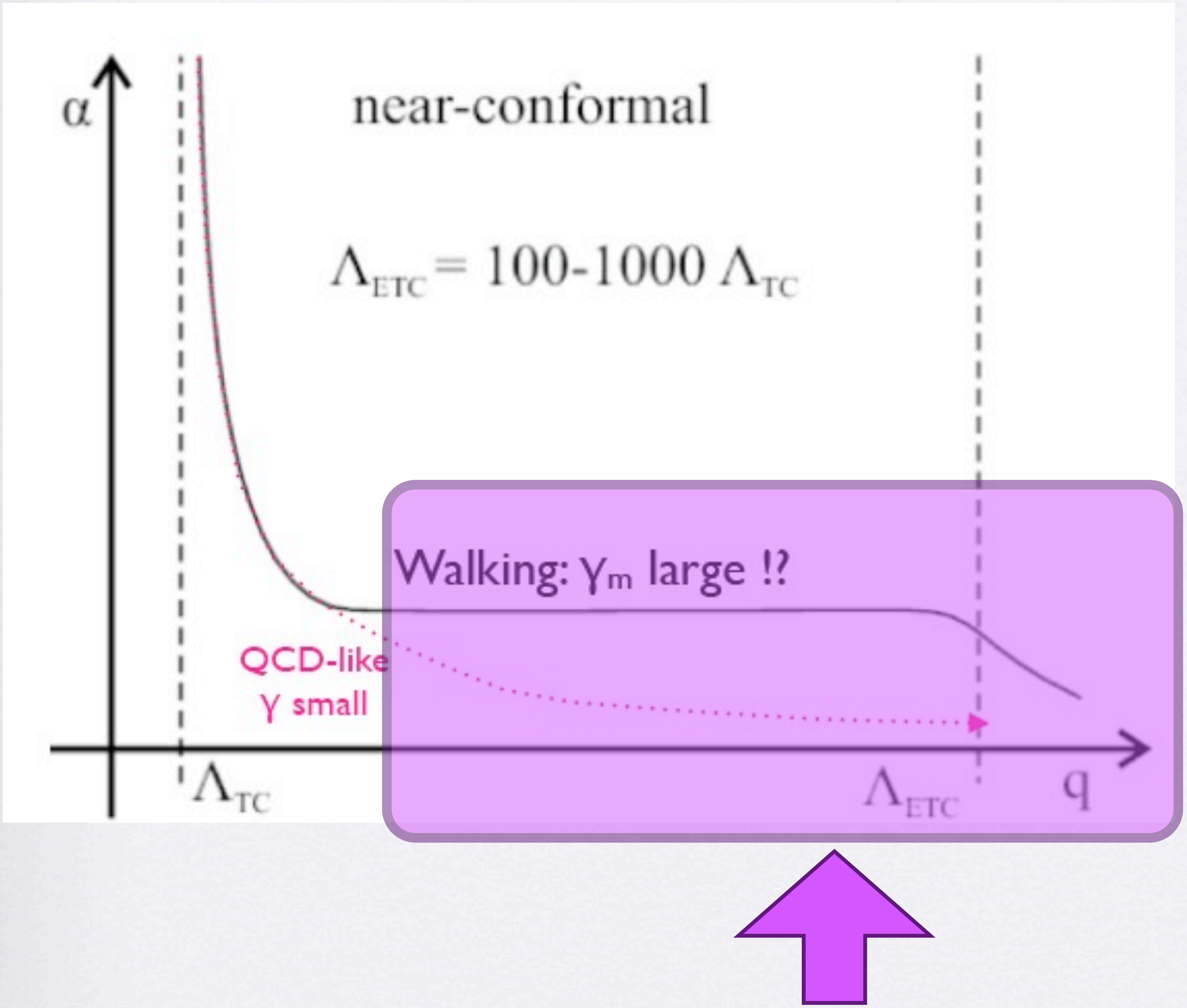
Large anomalous dimension for composite state

2-quark and 2-techni-fermion

$$\frac{c_2}{\Lambda_{ETC}^2}\langle\bar{\Psi}\Psi\rangle_{ETC}(\bar{\psi}\psi)$$

$$M_q \sim \frac{1}{\Lambda_{ETC}^2}\left(\frac{\Lambda_{ETC}}{\Lambda_{TC}}\right)^{\gamma^*}\langle\bar{\Psi}\Psi\rangle_{ETC}$$

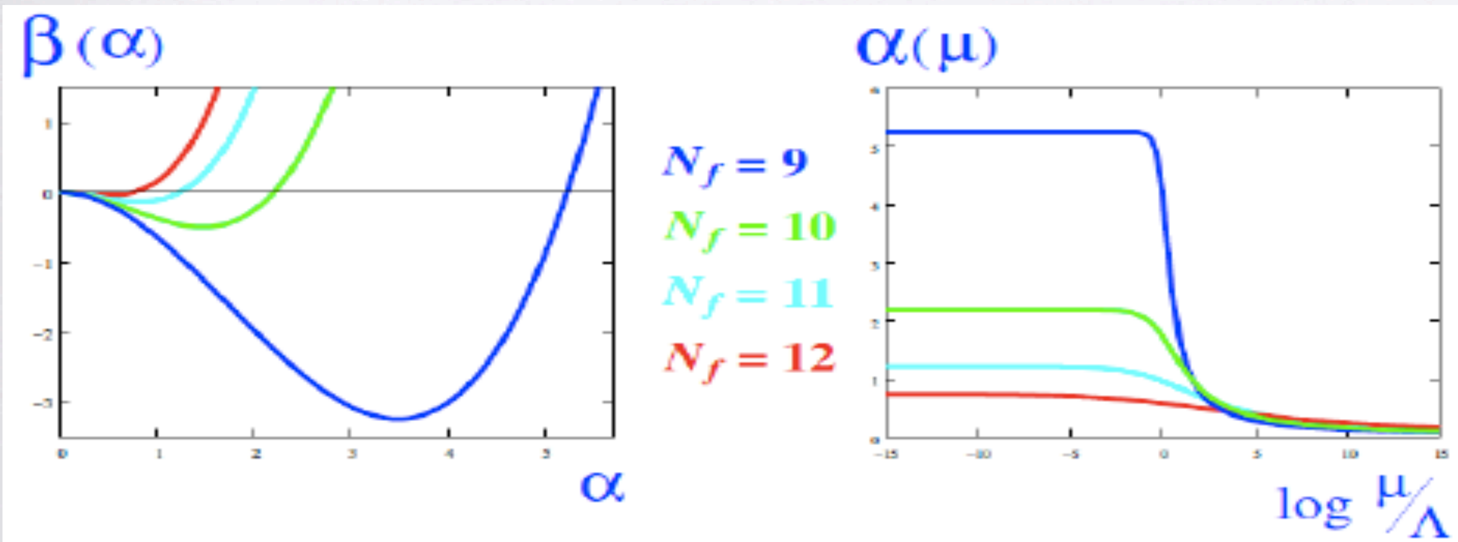
Is there a theory whose coupling constant show the behavior?




This part may be realized by
many flavor gauge theory.

# SU(3) Nf=12 theory

Two loop analysis




**perturbative (MS bar scheme)**

| | 2-loop | 3-loop | 4-loop |
|---|---|---|---|
| (alpha) | 0.75 | 0.44 | 0.47 |
| (g^2) | 9.4 | 5.5 | 5.9 |

T.A.Ryttov and R.Shrock,
Phys.Rev.D83,056011 (2011)

**S-D eq. with large Nc**

$$N_f^{cr} = 11.9$$

**Exact RG**

$$N_f^{cr} = 10.0^{+1.6}_{-0.7}$$

H.Gies and J.Jaeckel,
Eur.Phys.J. G46:433-438,2006

**Exact RG (+ 4 fermi interaction)**

$$N_f^{cr} = 11.58$$

Y.Kusafuka and H.Terao,
arXiv:1104.3606 [hep-ph]

Phase structure based on two loop

# Is there an IR fixed point in SU(3) Nf=12 theory?

Iwasaki et al, '04 ’13 (phase structure, correlation fn.)

Appelquist, Fleming, Neil '07, '09, '11 (running coupling, mass spectrum)

Deuzeman, Lombardo, Pallante, Miura '09, '11(finite temperature)

A. Hasenfratz '09, '10 (MCRG, phase structure)

DeGrand '11 (mass spectrum)

LatKMI ’12 (mass spectrum)

Fodor et al. '09 , '11(running coupling, phase structure, spectrum)

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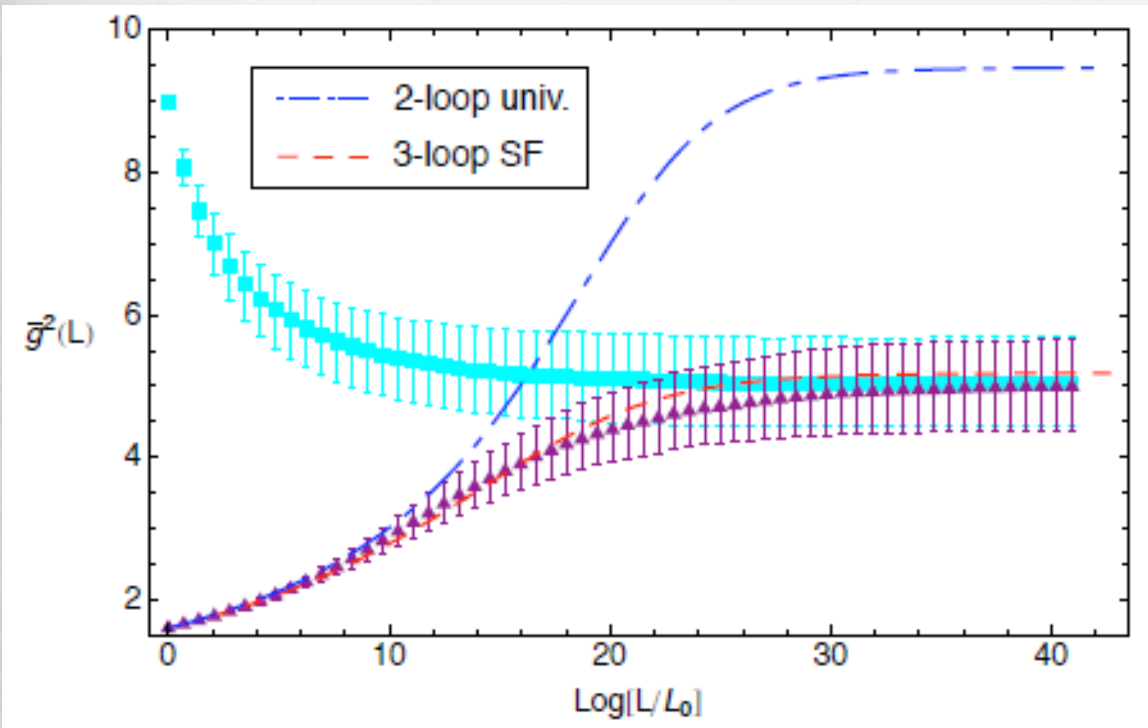
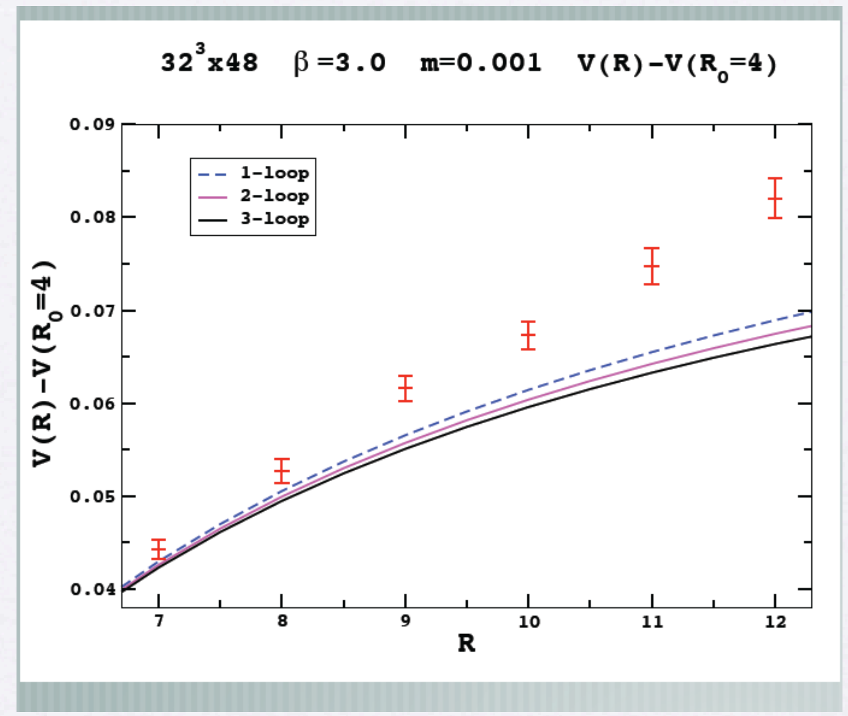
# Why there are controversial results?

# Running coupling constant

Appelquist et al. (SF scheme)
Phys.Rev.D79:076010,2009

Fodor et al. (potential scheme)
PoS LAT2009:055,2009,
talk at Lattice2010




Plot: Slide of K.Holland's talk at Lattice2010

## Relationship between two renormalization schemes

scheme transformation: $g_1 \to g_2 = f(g_1)$

beta fn. $$\beta(g_2) = \frac{\partial f(g_1)}{\partial g_1}\beta(g_1)$$

Existence of fixed point is scheme independent.

2013年3月6日水曜日

# Why there are controversial results?

The continuum extrapolation should be taken carefully.

- conformal theory is realized in the continuum
- Lattice law data suffer from O(a)
- O(a) effect is renormalization scheme dependent

In the study on the phase structure, parameter search is not enough?

- tuning the beta value

# chiral symmetry

Z.Fodor et al.: Phys.Lett.B703:348-358,2011.

measured mass spectrum and chiral condensate at beta=2.2
in several lattice sizes and fermion bare masses.

**- Fodor et. al. comparison between two hypotheses**

chiral extrapolation using conformal hypothesis does not work.
chiral broken hypothesis works well.
chiral symmetry is weakly broken

fit fn: $$\langle \bar{\Psi}\Psi \rangle = c_0 + c_1 m + c_2 m^2$$

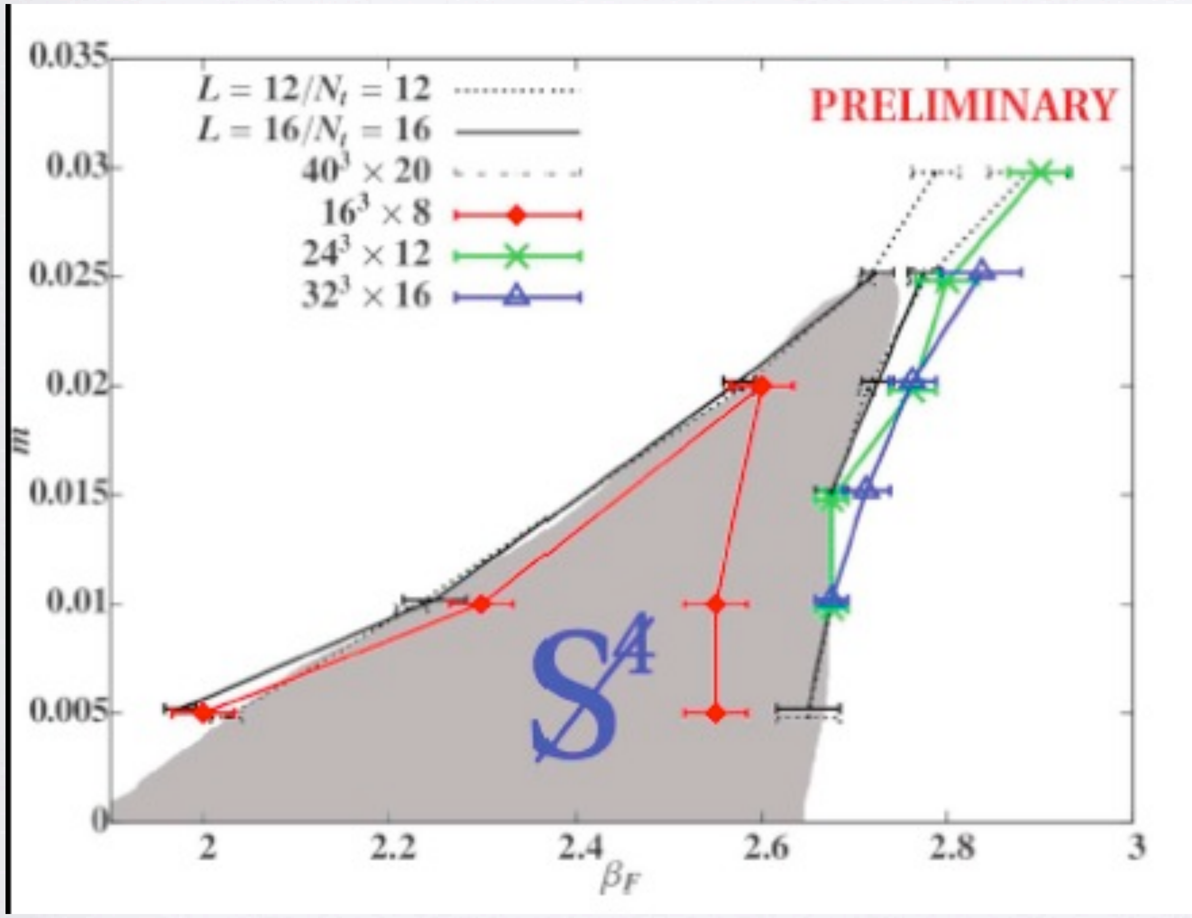
$$c_0 = 0.00282 \pm 0.00021$$

**- LSD collab. fit the limited # of Fodor's data using conformal hypotheses** PRD84(2011)054501

**- DeGrand considers finite-scaling** PRD84 (2011) 116901

conformal hypothesis also works well.

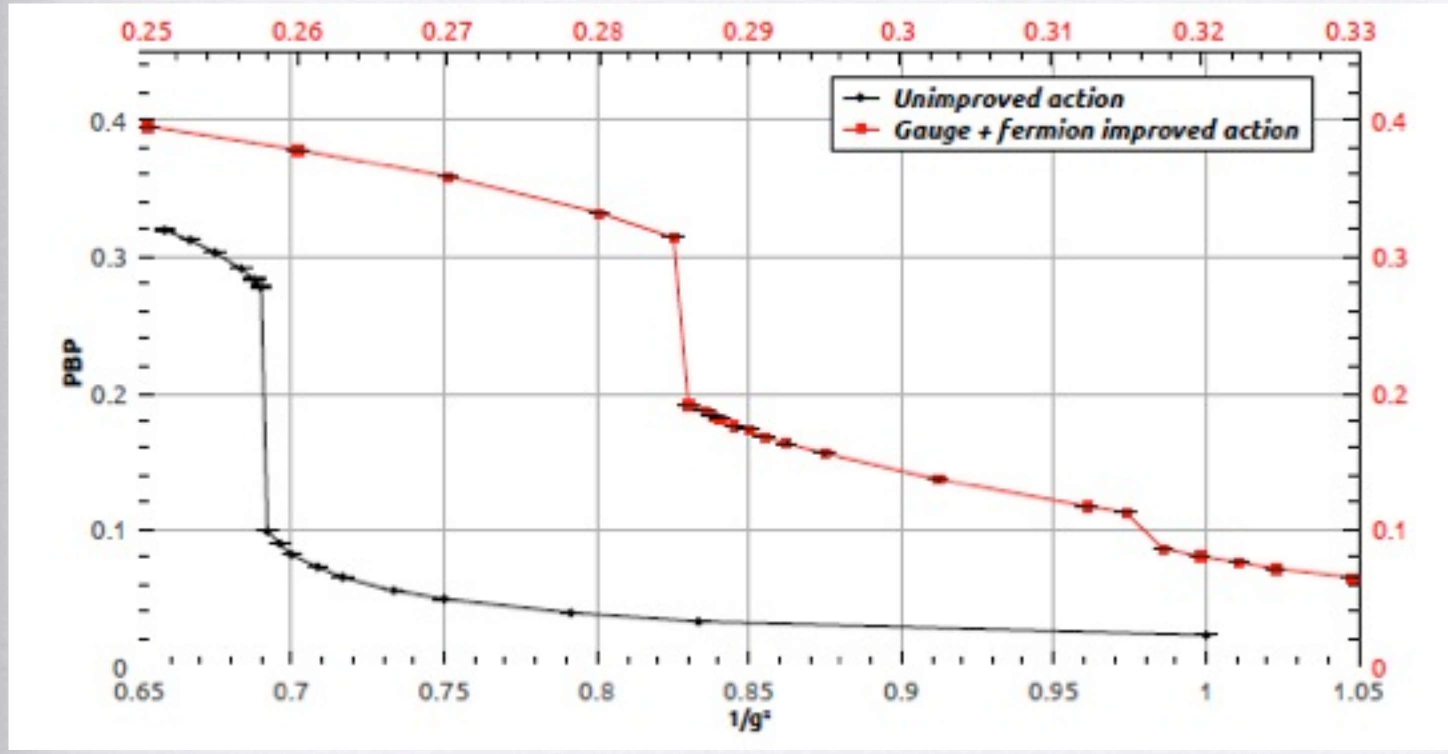
# Weakly chiral symmetry broken phase




Chang, Hasenfratz and Schaich:
Phys.Rev.D85 (2012) 094509

HYP smearing

two jumps of chiral condensate and in the intermediate region shift symmetry is broken.



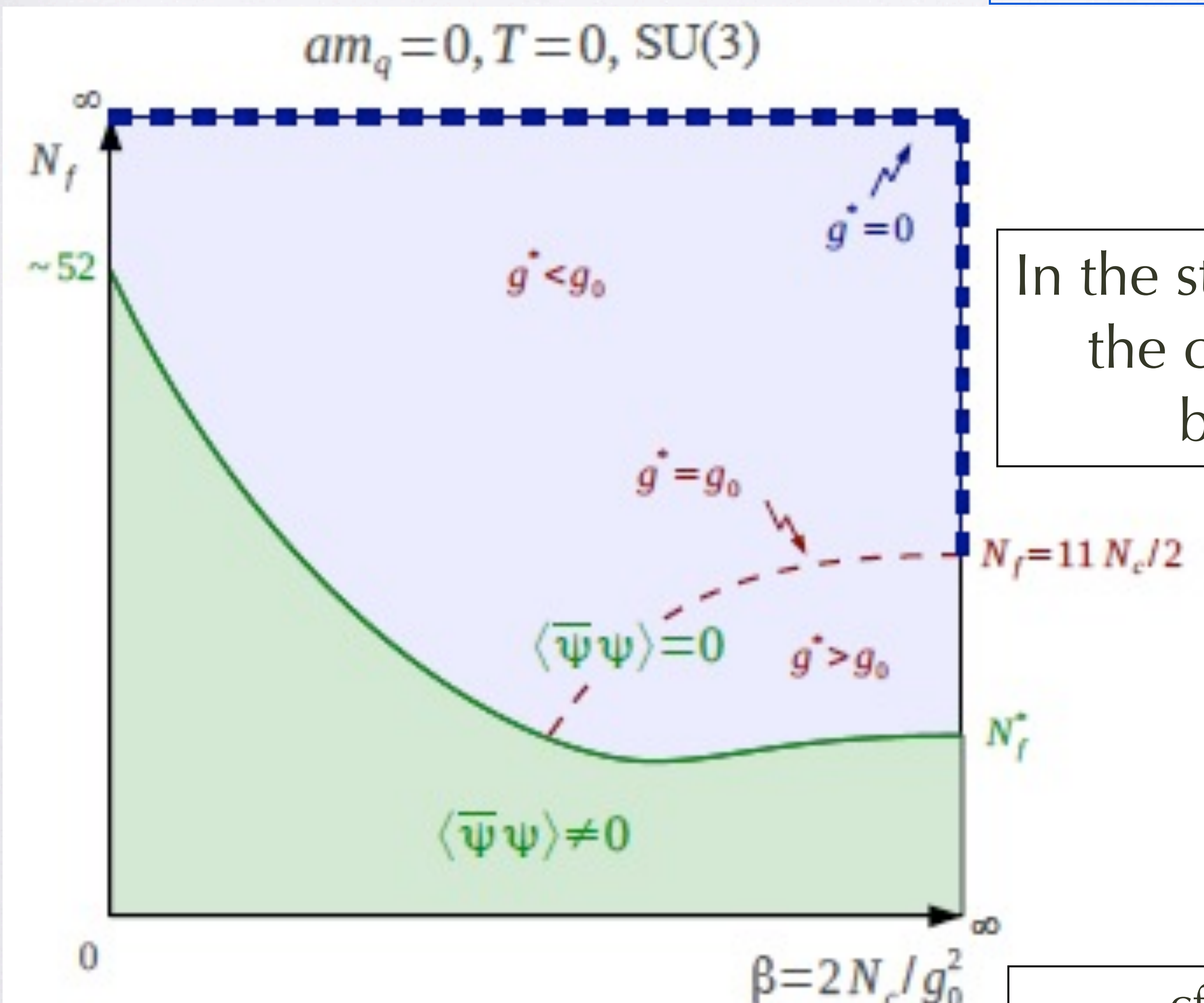

Deuzeman, Lombardo, da Silva and Pallante:
arXiv: 1209.5720

Naik improvement

next-to-nearest neighbor terms are no longer irrelevant and indeed modify the pattern observed without improvement.

conjectured phase diagram for many flavor SU(3) gauge theory

de Forcrand, Kim and Unger:
arXiv:1208.2148



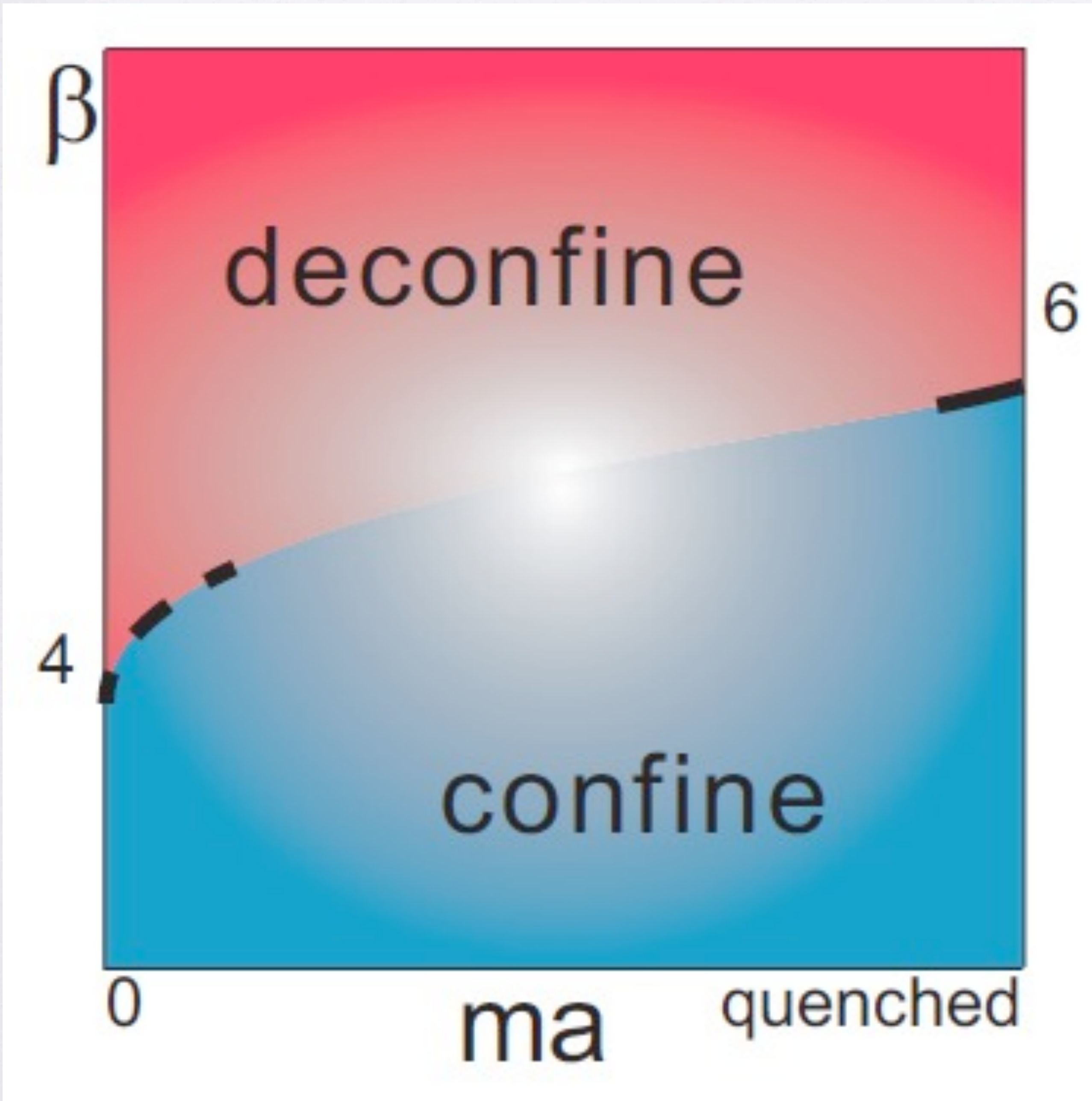

In the strong coupling limit, the chiral symmetry is broken Nf<52.

cf. for SU(Nc),
Tomboulis arXiv:1211.4842

# Our result

arXiv:1212.1353 [hep-lat]

# Phase diagram for SU(3) Nf=12 naive staggered fermion with the twisted boundary condition.




In the above phase,

$$\langle |P_t| \rangle \neq 0$$

In the bottom phase,

$$\langle |P_t| \rangle \simeq 0$$

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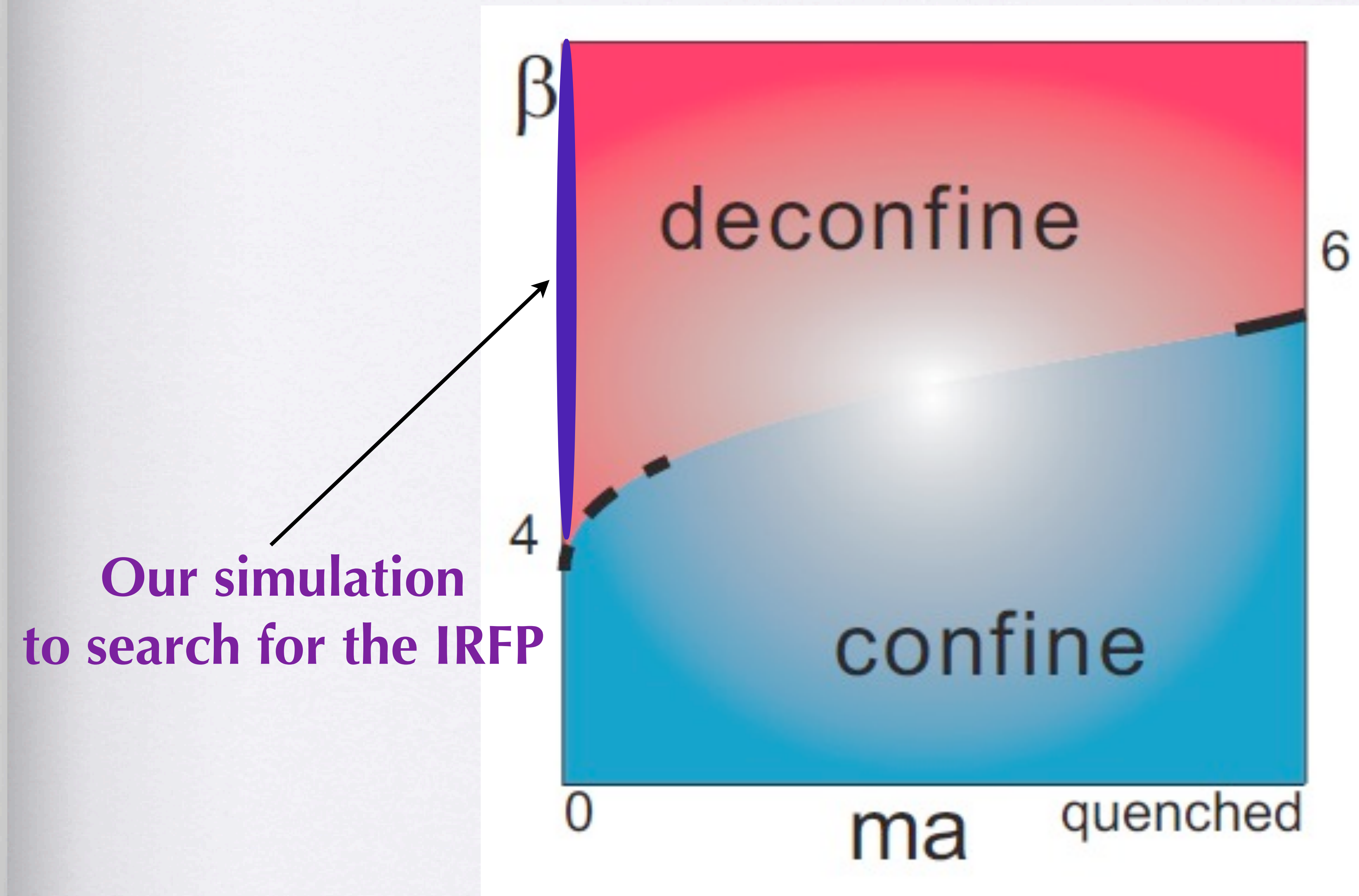


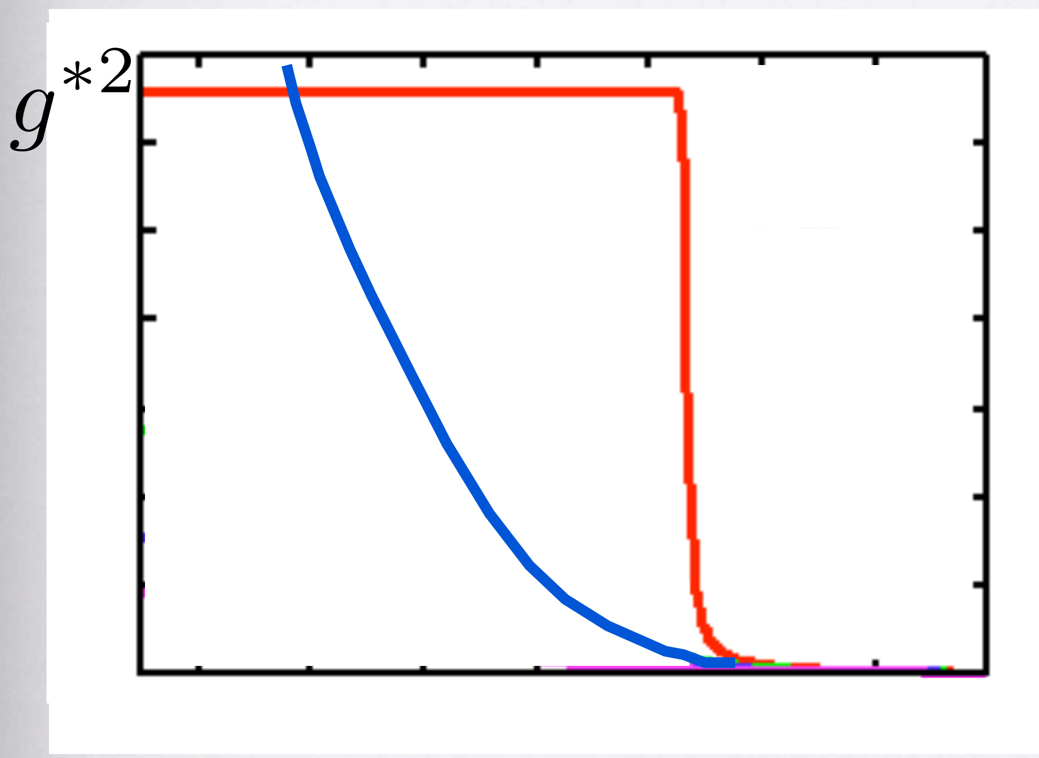

We also see that the chiral symmetry is preserved in this region.

# Running coupling

# Measure the growth ratio

running coupling constant

$g_R^2$

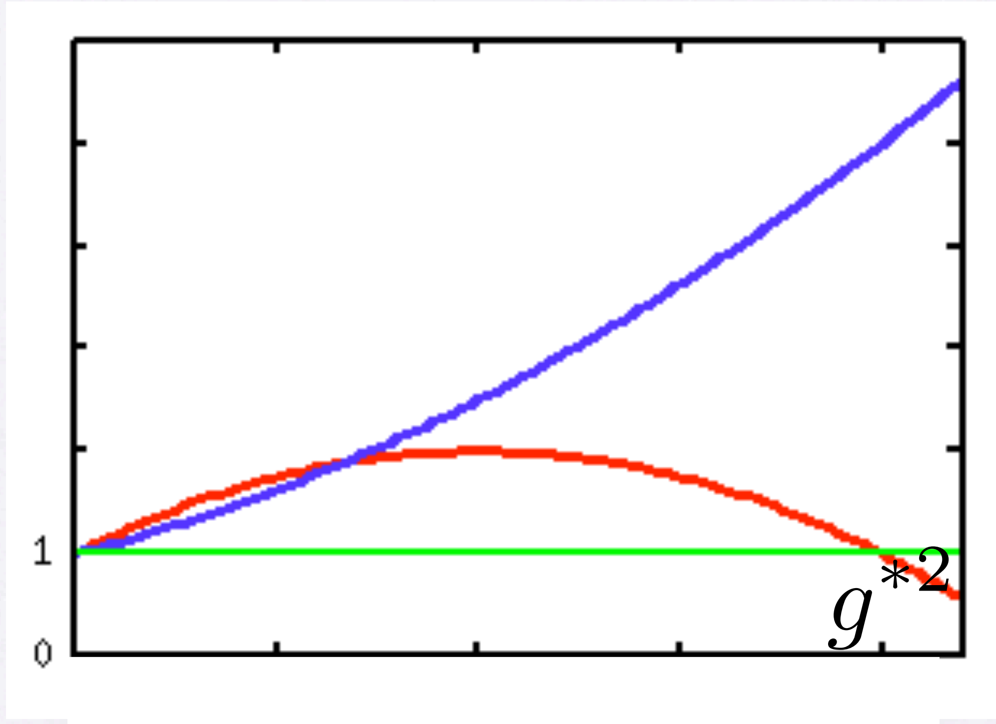



$\ln(L_0/L)$

systematic error is accumulated

growth ratio

$\sigma(u)/u = g_R^2(1/sL)/g_R^2(1/L)$




$g_R^2(\mu = 1/L)$

systematic error is not accumulated

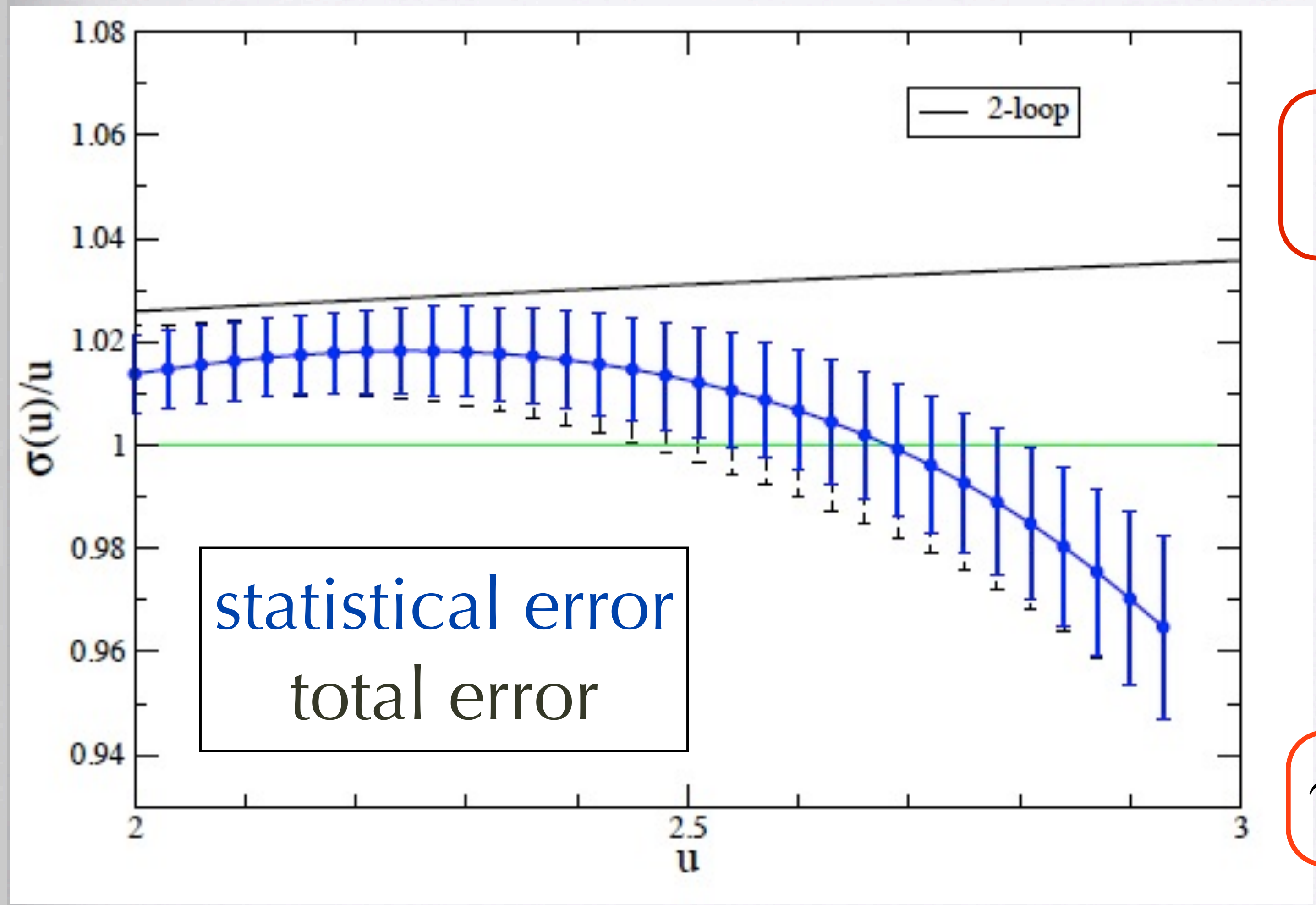
# No O(a) renormalization scheme

Twisted Polyakov loop (TPL) scheme

de Divitiis et. al. NPB422(1994)382

$$g_{TP}^2 = \frac{1}{k}\frac{\langle \sum_{y,z} P_1(y,z,L/2a)P_1(0,0,0)^* \rangle}{\langle \sum_{x,y} P_3(x,y,L/2a)P_3(0,0,0)^* \rangle}$$

# Growth ratio of TPL coupling
# (local fit analysis)




$$g^{*2}_{\mathrm{TPL}} = 2.69 \pm 0.14(\mathrm{stat.})^{0}_{-0.16}(\mathrm{syst.})$$

Around the fixed point, the beta fn. can be approximated by the linear fn.

$$\beta(g^2) \sim \gamma_g^*(g^{2*} - g^2)$$

Our result

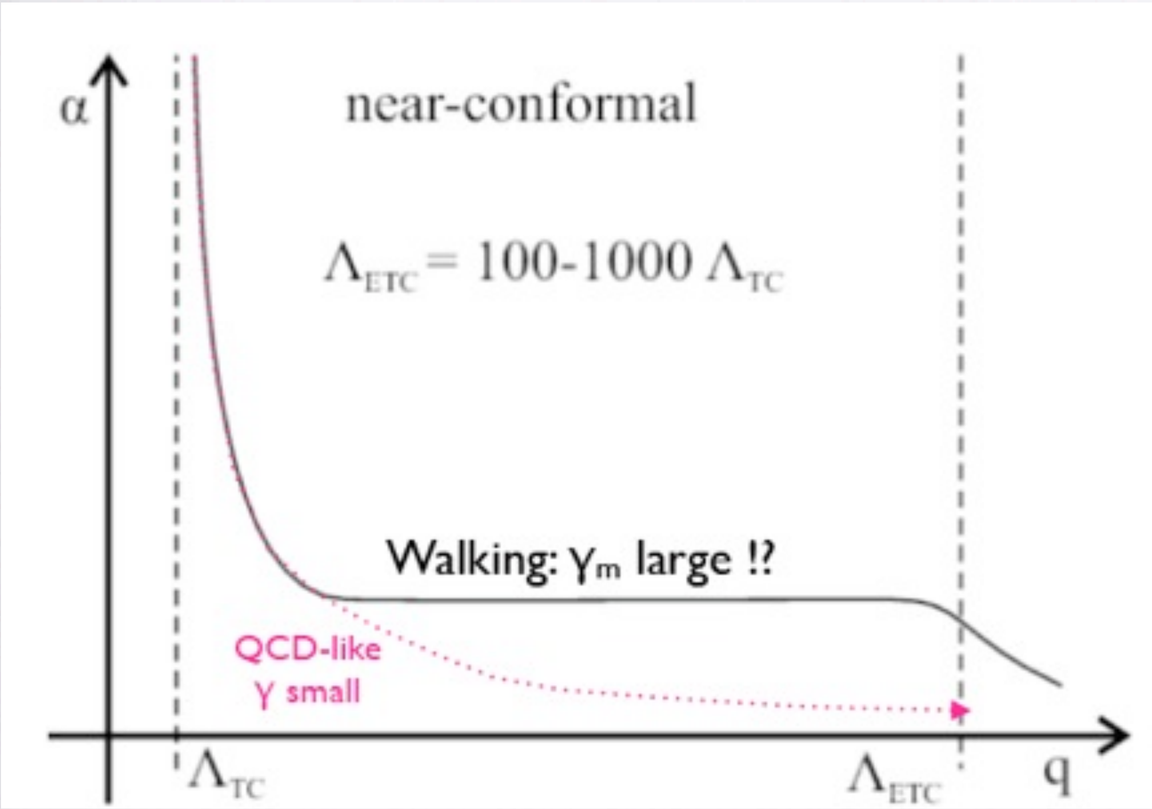
$$\gamma_g^* = 0.57^{+0.35}_{-0.31}(\mathrm{stat.})^{0}_{-0.16}(\mathrm{syst.})$$

| SF scheme | 2 loop at $g^{2*} = 9.4$ | 4 loop (MS bar) |
|---|---|---|
| $\gamma_g^* = 0.13 \pm 0.03$ | $\gamma_g^* = 0.36$ | $\gamma_g^* = 0.28$ |

# Anomalous dimension

## —Preliminary result—

# The other important critical exponent around the IRFP




2-quark and 2-techni-fermion

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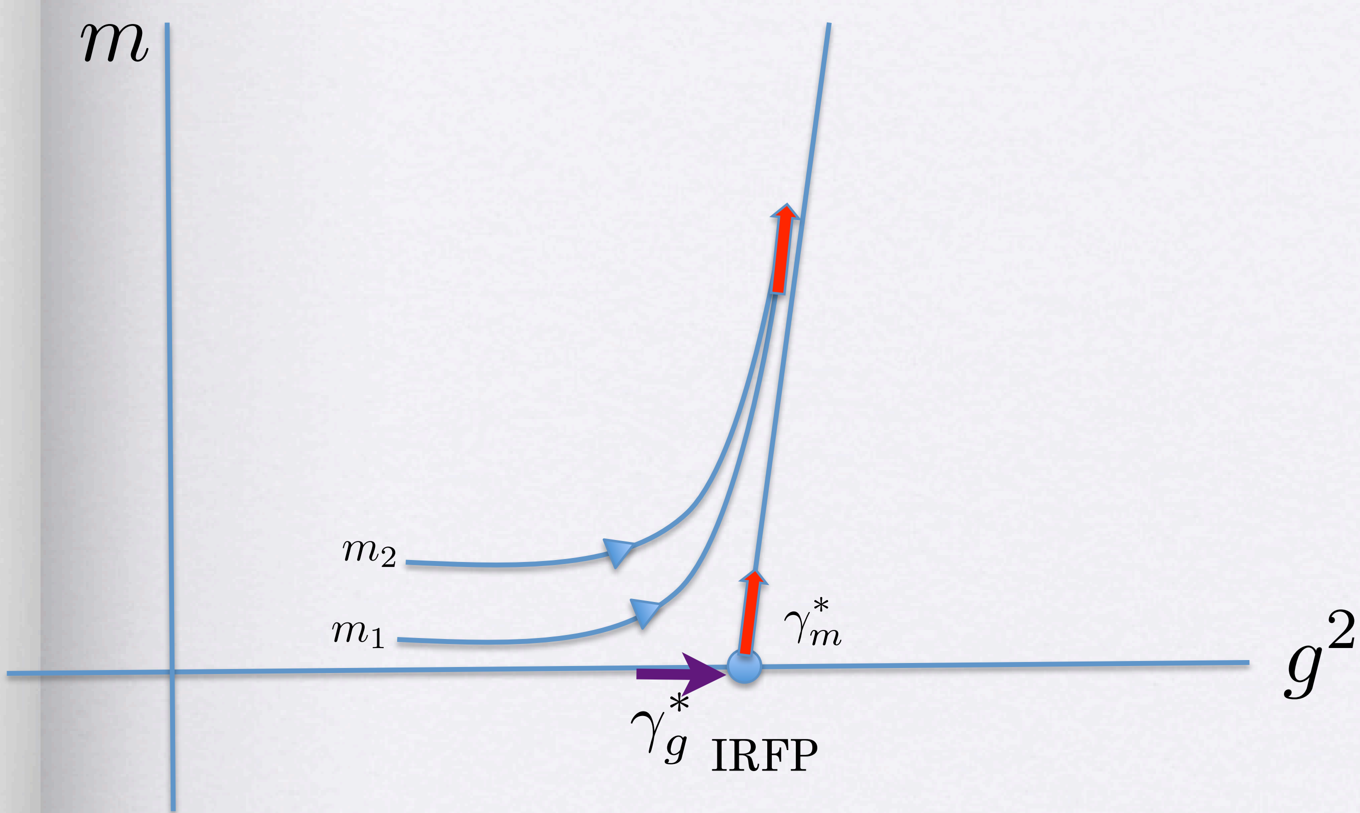



2loop prediction:

$$\gamma_g^* = 0.36 \qquad \gamma_m^* = 0.77$$

# Mass anomalous dim
# from the several studies

2 loop

3 loop (MS bar)

4 loop (MS bar)

Schwinger-Dyson
(ladder)

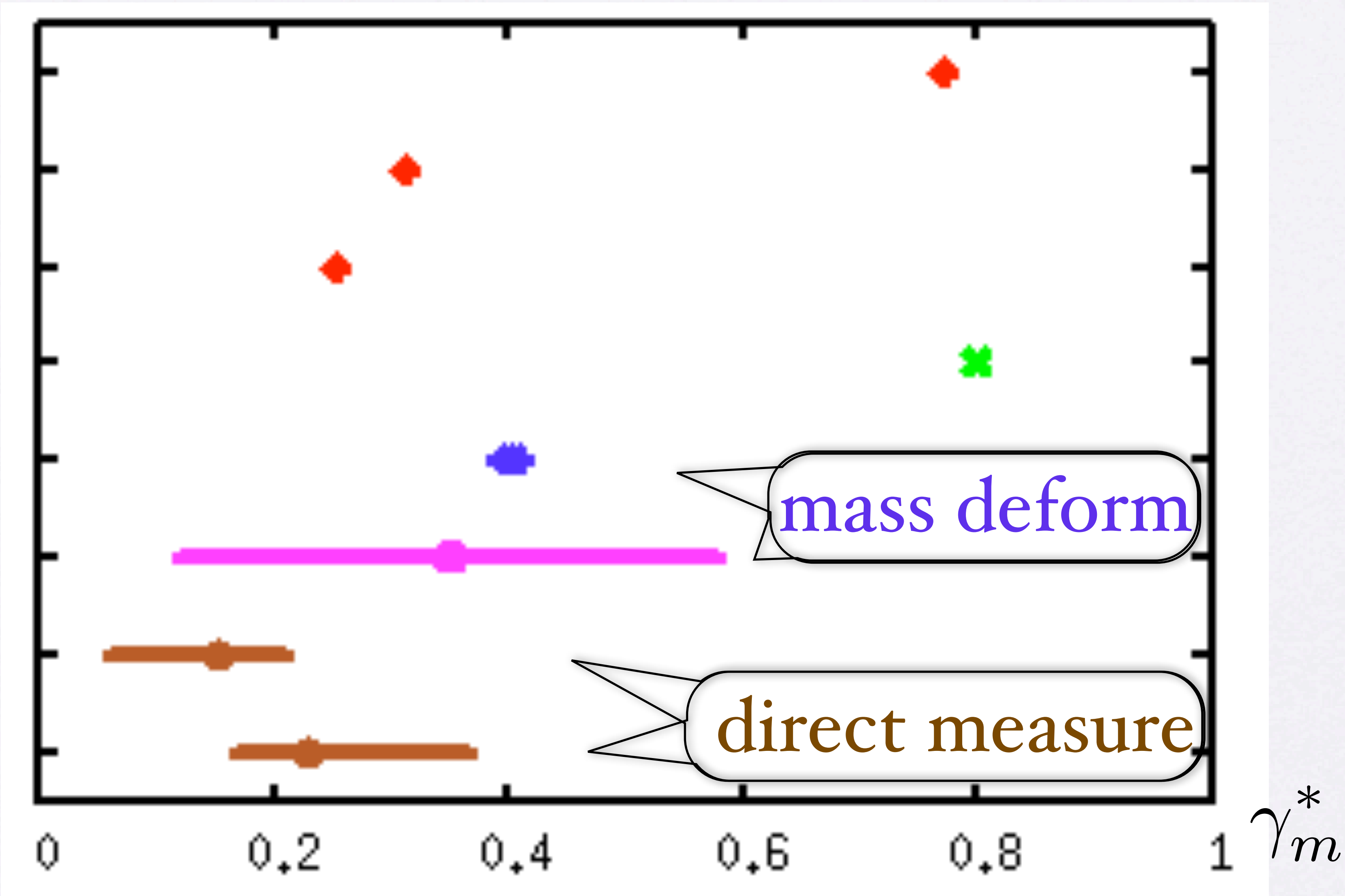
Appelquist et.al

DeGrand

Our result (r=1/3)

(r=1/4 w. stat. err.)

(preliminary)




Vermaseren, Larin and Ritbergen PL B405(1997)327

Ryttov and Shrock PRD83 (2011) 056011

Yamawaki, Bando and Matumoto: PRL 56, 1335 (1986)

PRD84(2011)054501

PRD84(2011)116901

# correlation fn. of nearly conformal theory

Ishikawa, Iwasaki, Nakayama and Yoshie: arXiv:1301.4785

two point fn. of a meson state

$$G_H(t) = \sum_x \langle \bar{\psi}\gamma_H\psi(x,t)\bar{\psi}\gamma_H\psi(0)\rangle$$

conformal theory (massless, continuum)

$$G_H(t) = \tilde{c}\,\frac{1}{t^{\alpha_H}}$$

$$\alpha_H = 3 - 2\gamma^*$$

confined theory

$$G_H(t) = c_H\ \exp(-m_H t)$$

(nearly) conformal theory

$$G_H(t) = \tilde{c}_H\,\frac{\exp(-\tilde{m}_H t)}{t^{\alpha_H}}$$

Yukawa-type fit fn works well.

cf. 2-dim. scalar CFT on a cylinder

$$\langle\phi(z)\phi(0)\rangle = \frac{1}{z^{2h}} \quad \Rightarrow \quad \langle\phi(w)\phi(0)\rangle = \left(\frac{2\pi}{L}\right)^{2h}(2\sinh[\pi w/L])^{2h}$$

# Conclusion

標準模型を超える物理を考える時代。

walking technicolor模型には、非摂動論領域の精密な情報が必要。

conformal理論=QCDの赤外領域と異なる理論

- conformal理論の存在の判定、方法論の確立
- 格子理論の相構造、 有限体積の場合の性質...

我々の結果

SU(3) Nf=12 massless 理論には固定点が存在する

連続極限をとる事、パラメータサーチが重要

固定点の共形場の理論の性質、模型の低エネルギー物理への予言は

これから！