Numerical study on quantum entanglement entropy in 4d SU(3) gauge theories

Etsuko Itou (KEK)

collaboration with K. Nagata (KEK), Y.Nakagawa, A. Nakamura (Hiroshima U., RCNP) and V.I.Zakharov (Max Planck Inst.)

cf. arXiv:0911.2596 and 1104.1011:
Y.Nakagawa, A.Nakamura, S.Motoki and V.I.Zakharov

"10sor network workshop"
--- Field 2 x 5 joint workshop on new algorithms for quantum manybody problems —
@ Tokyo University, Future Center 2015/5/14

JICFuS

entanglement entropy

=?

a novel approach

to understand the confinement

Don’t think....feel....

Outline

- Introduction (QCD in 4d)
- Definition of entanglement entropy
- Replica method
- Results for the quenched QCD

QCD

dynamics of gluons and quarks

- Yang-Mills theory (SU(3)gauge theory)

  local symmetry

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \bar{\psi}(iD_\mu\gamma^\mu - m)\psi$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + igf^{abc}A^b_\mu A^c_\nu \qquad D_\mu = \partial_\mu - igA^a_\mu t^a$$

- parameter: gauge coupling, fermion mass

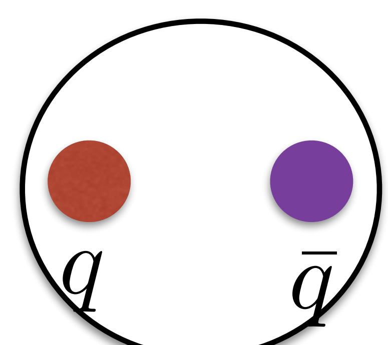
- adjoint reps. of SU(3)
  (gluon: messenger of the force)

$$A^a_\mu(x) \quad \begin{cases} a = 1, \cdots, 8 \\ \mu = 1, \cdots, 4 \end{cases}$$

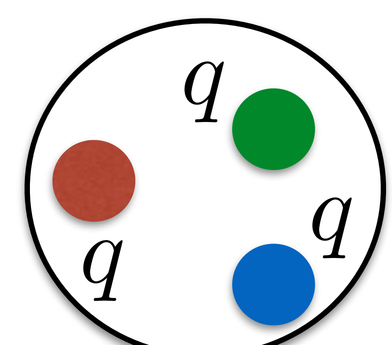
- fundamental reps. of SU(3)

$$\psi^i_\alpha(x), \bar{\psi}^{\bar{i}}_\alpha \quad \text{or } q, \bar{q} \quad \begin{cases} i = 1, 2, 3 \\ \alpha = 1, \cdots, 4 \end{cases}$$

  (quark: fundamental element of matter)



meson
ex)pion

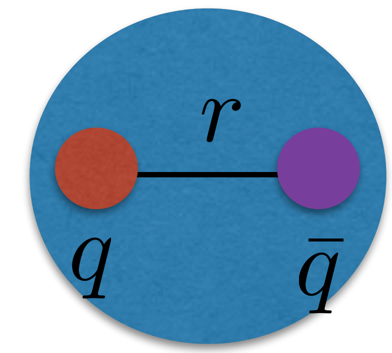


baryon
ex)proton

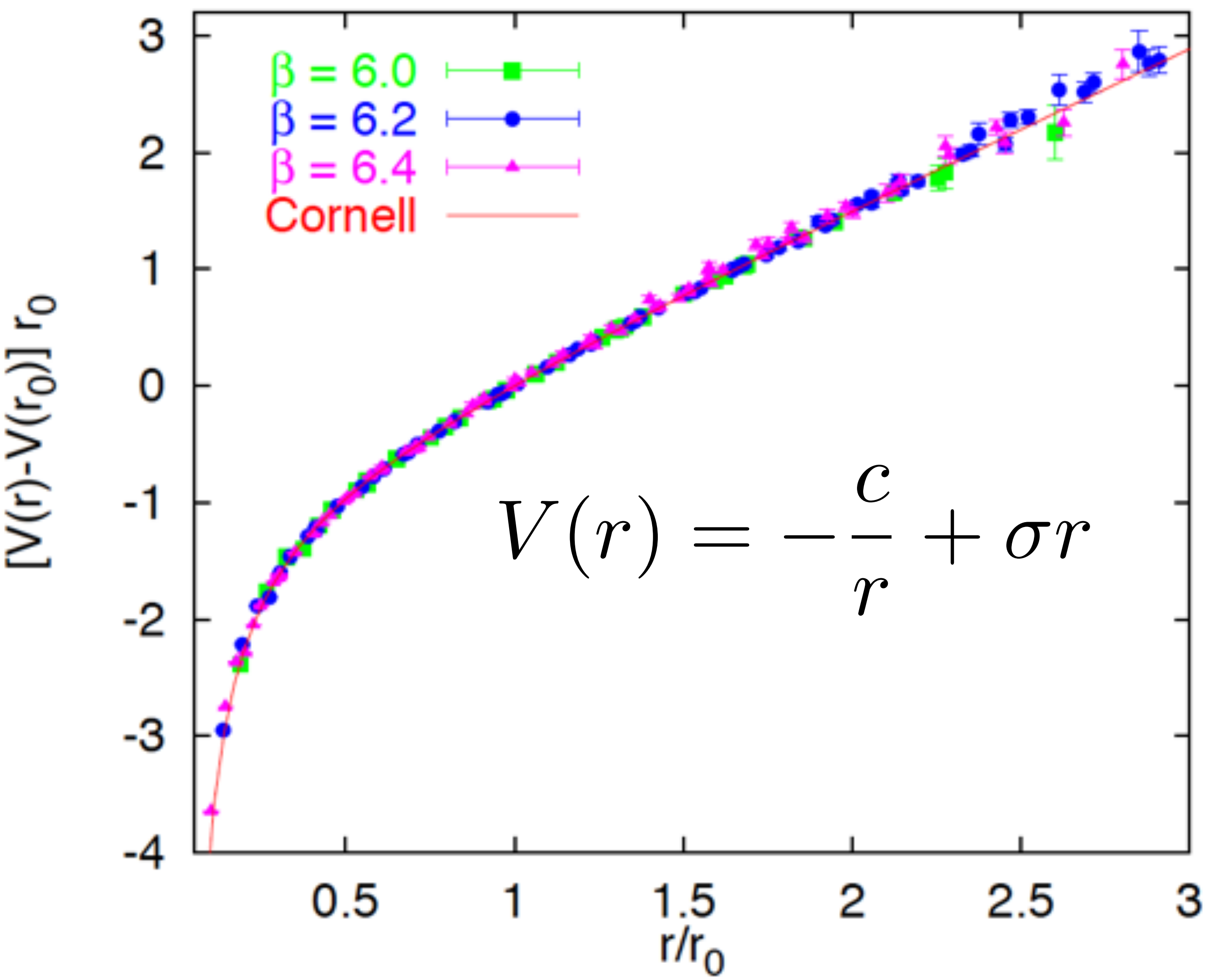
remarks: pure YM or quenched QCD => only first term full QCD => whole lagrangian

confinement

A nonperturbative property of QCD



Hadron

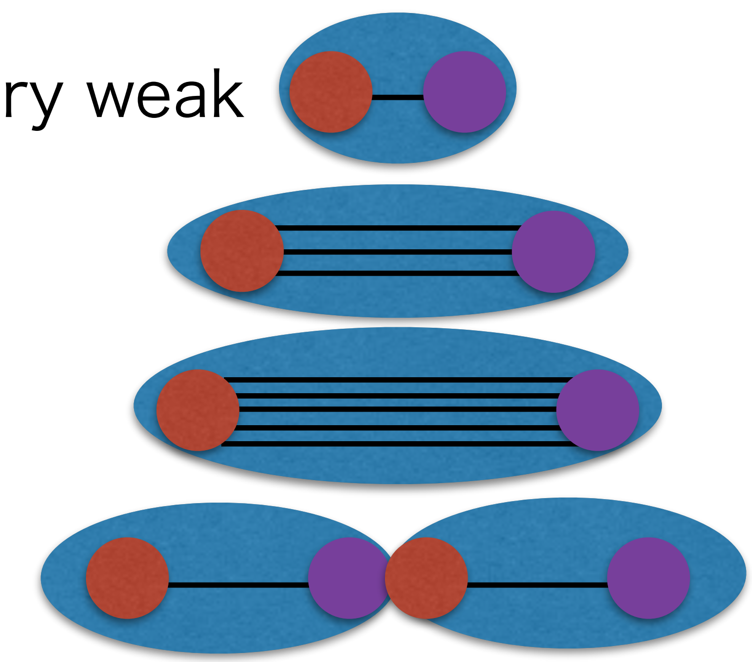


$$V(r) = -\frac{c}{r} + \sigma r$$

G. Bali Phys. Rept.343,(2001)1

interaction is very weak

$r > \Lambda_{\rm QCD}^{-1}$



- q-qbar potential
- center symmetry (only for pure YM)

Basic properties of E.E.

entanglement entropy for quantum system

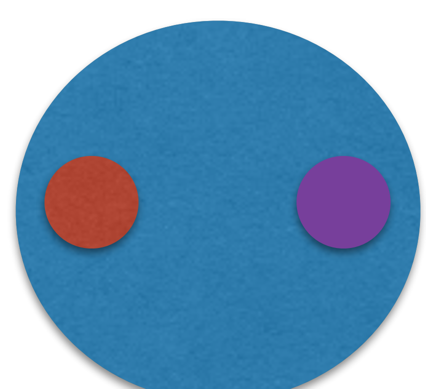
- how much a quantum state is entangled quantum mechanically
- d.o.f of the system
- quantum properties of the ground state for the system
- in finite T system, it gives a thermal entropy

QCD theory (T=0)

A color confinement changes the d.o.f of the system

microscopically $\Lambda_{\rm QCD}$ macroscopically

colorful (gluons) $\sim O(N_c^2)$



colorless (singlet) $\sim O(1)$

Definition of the entanglement entropy

Entanglement entropy (E.E.)

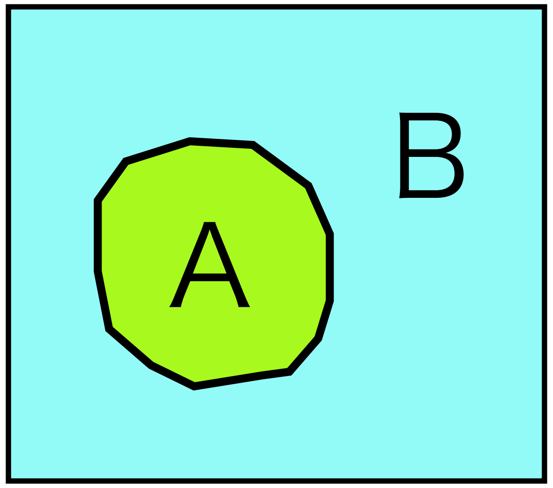
von Neumann entropy $$S_{tot} = -\mathrm{Tr}\rho_{tot} \log \rho_{tot}$$

density matrix $$\rho_{tot} = |\Psi\rangle\langle\Psi|$$ $|\Psi\rangle$:pure ground state

decompose total Hilbert space into two subsystems $$\mathcal{H}_{tot} = \mathcal{H}_A \otimes \mathcal{H}_B$$

reduced density matrix $$\rho_A = -\mathrm{Tr}_{\mathcal{H}_B}\rho_{tot}$$

entanglement entropy $$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A$$

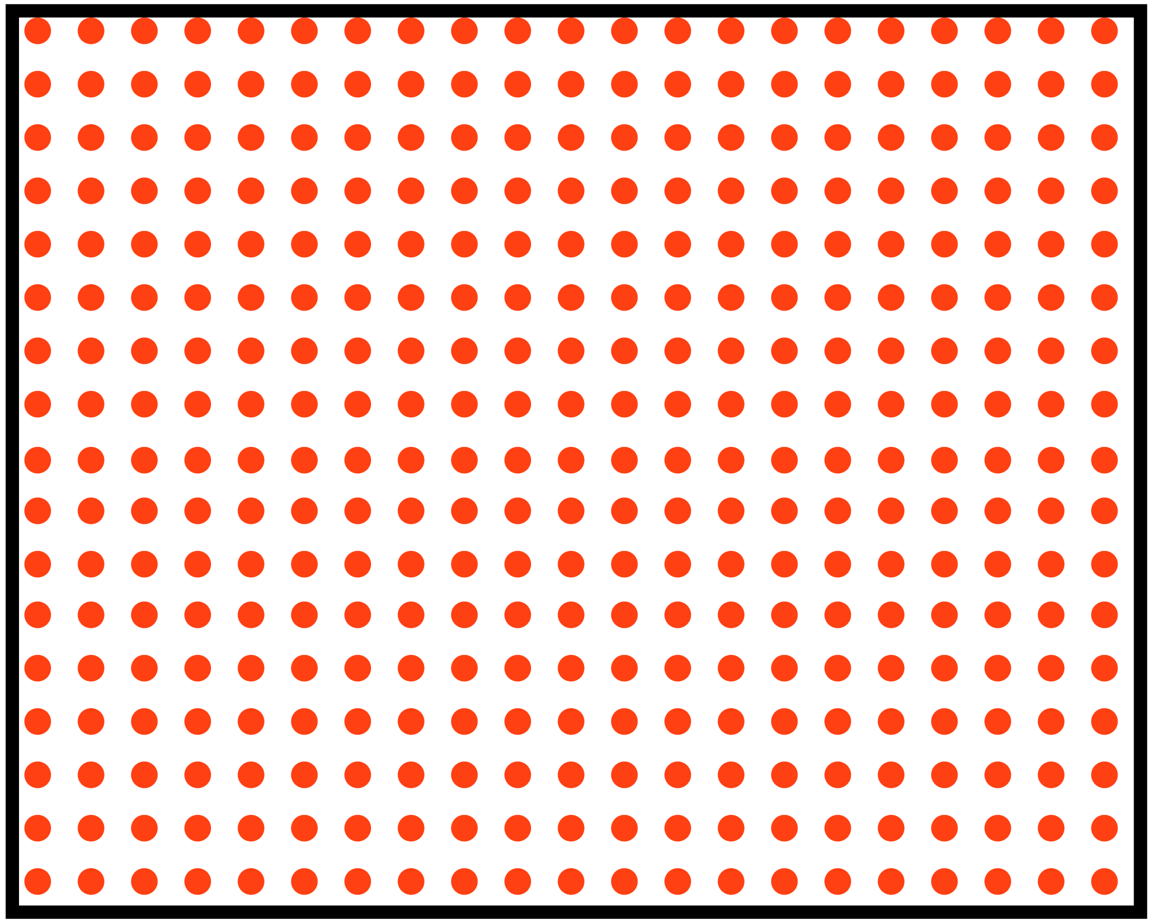


At finite T, it is equivalent to the thermal entropy.

Schematic picture of E.E.

Holzhey,Larsen and Wilczek: NPB424 (1994) 443
Calabrese and Cardy: J.S.M.0406(2004)P06002
Calabrese and Cardy, arXiv:0905.4013

(1+1)-dim. model



Schematic picture of E.E.

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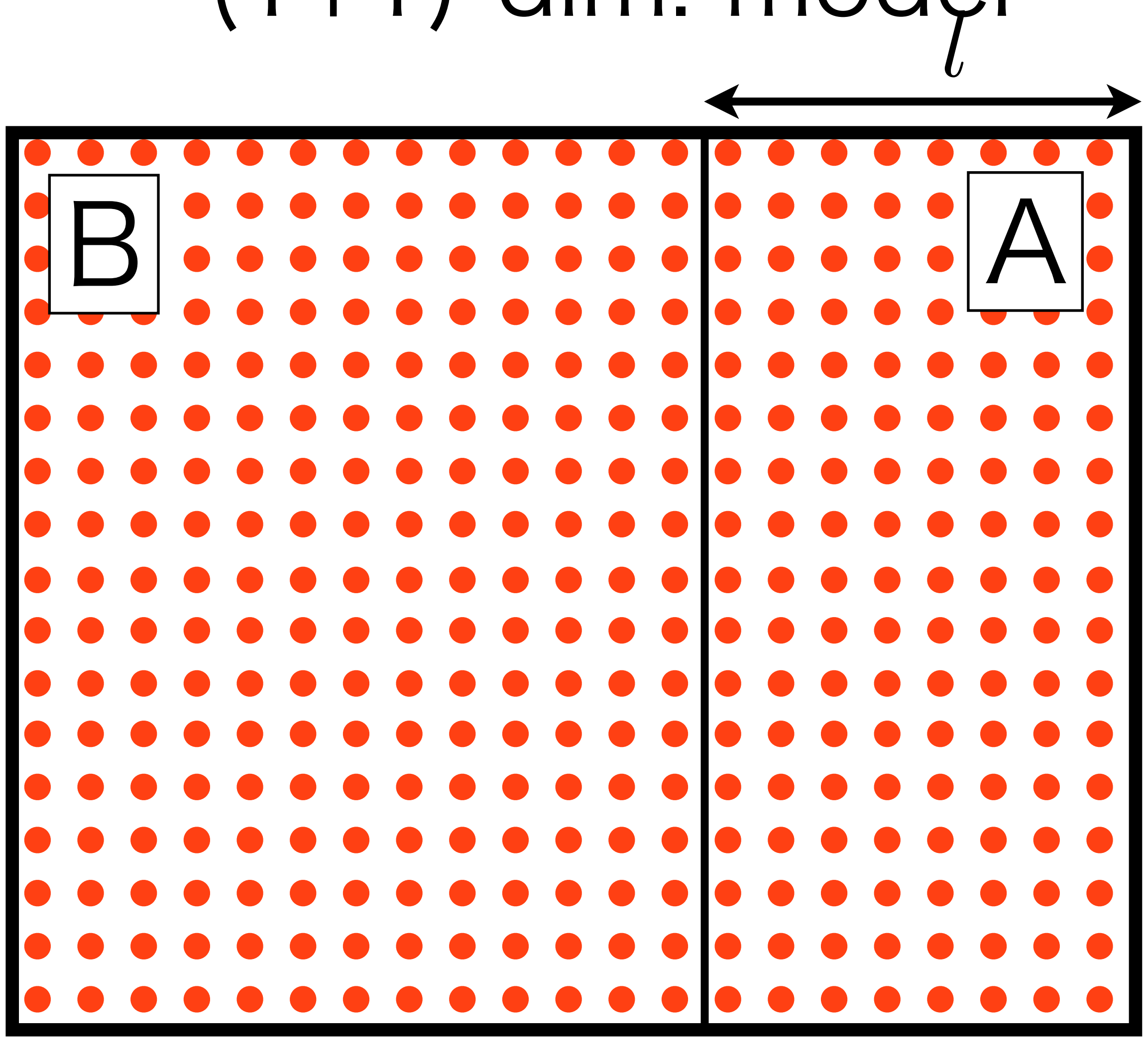

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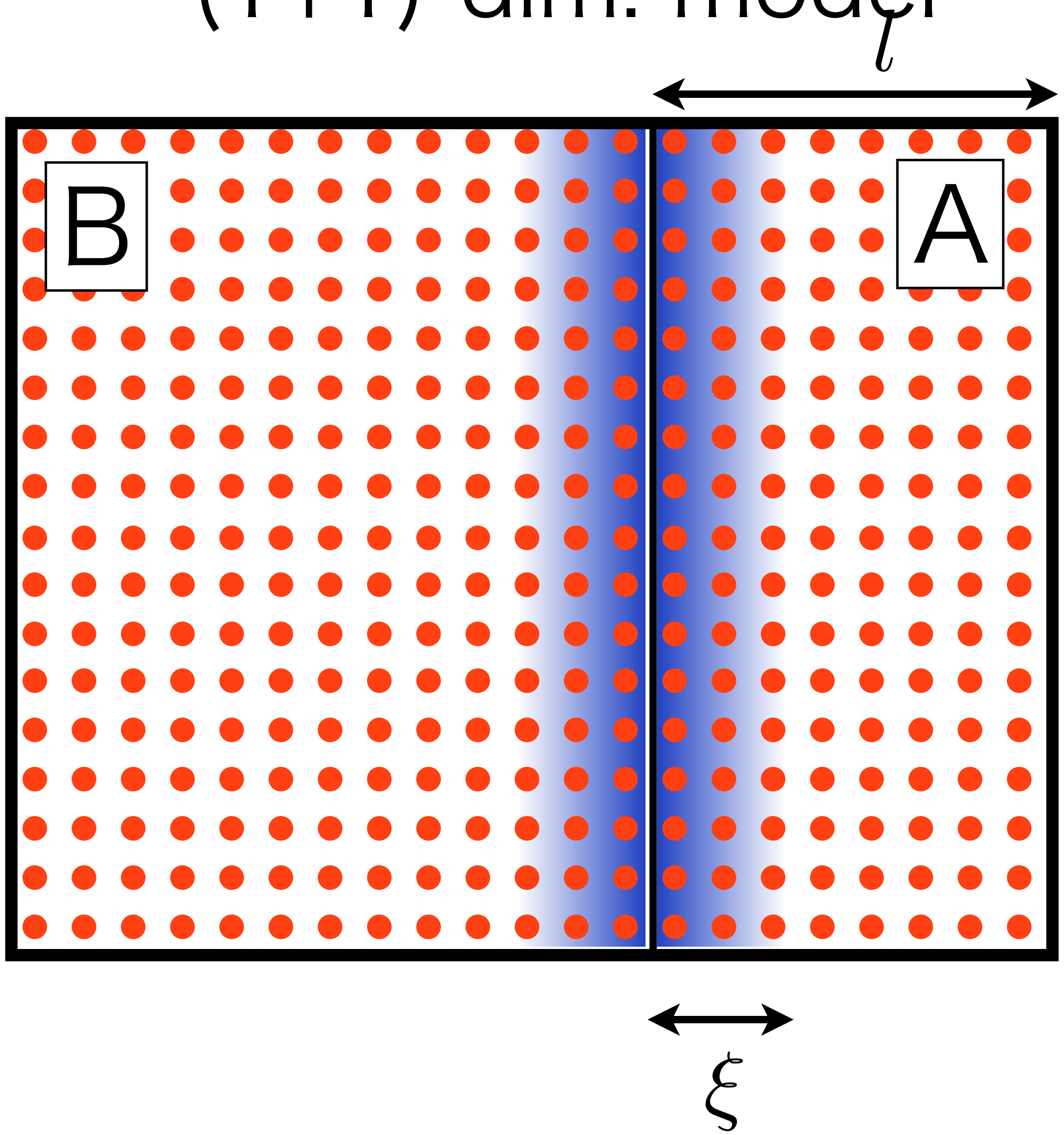


ξ: correlation length of the system

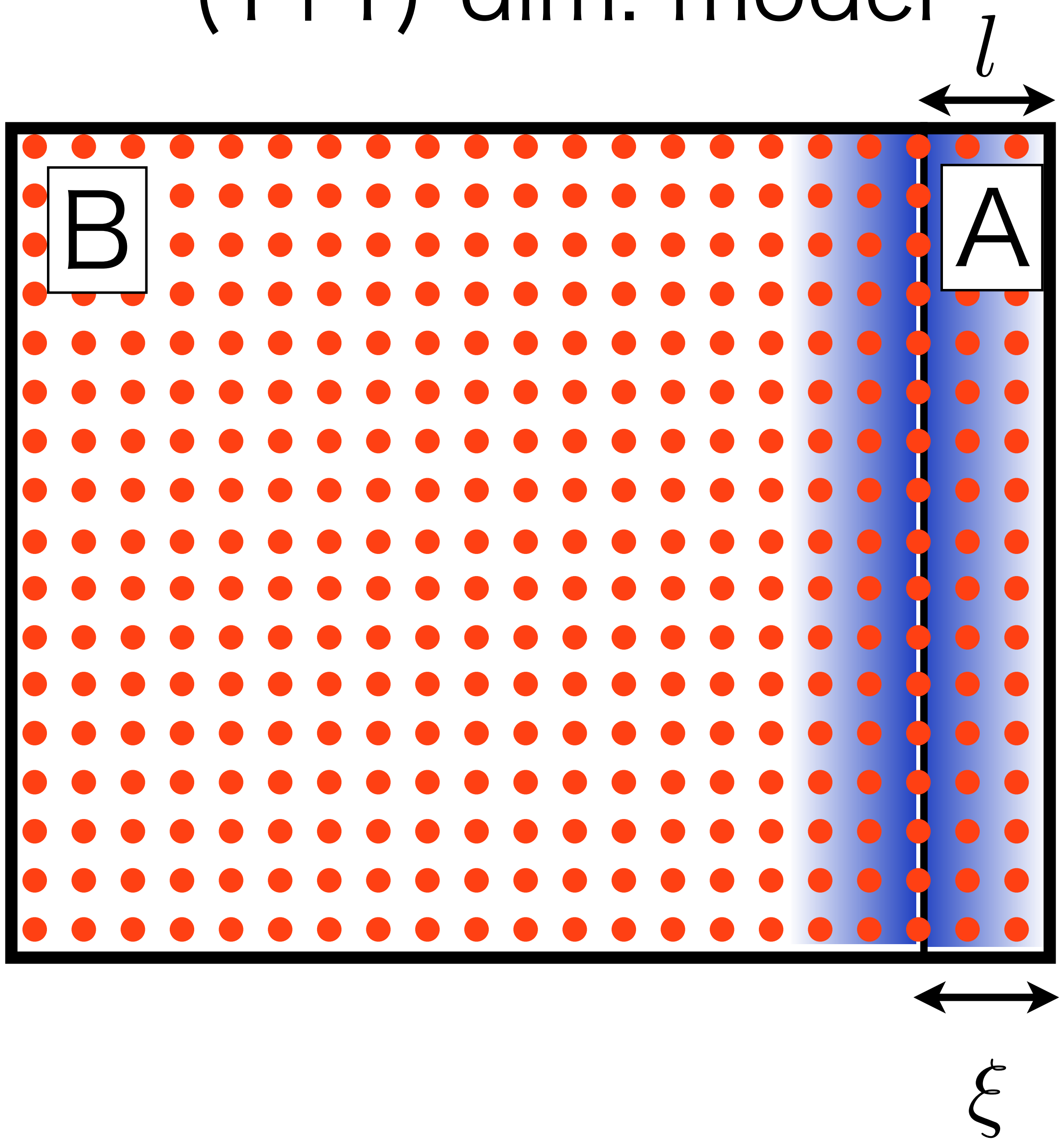
At the critical point,

$$S_A(l) = \frac{c}{3} \log \frac{l}{a} + c_1$$

c is the central charge in 2d CFT.

Schematic picture of E.E.

(1+1)-dim. model



ξ: correlation length of the system

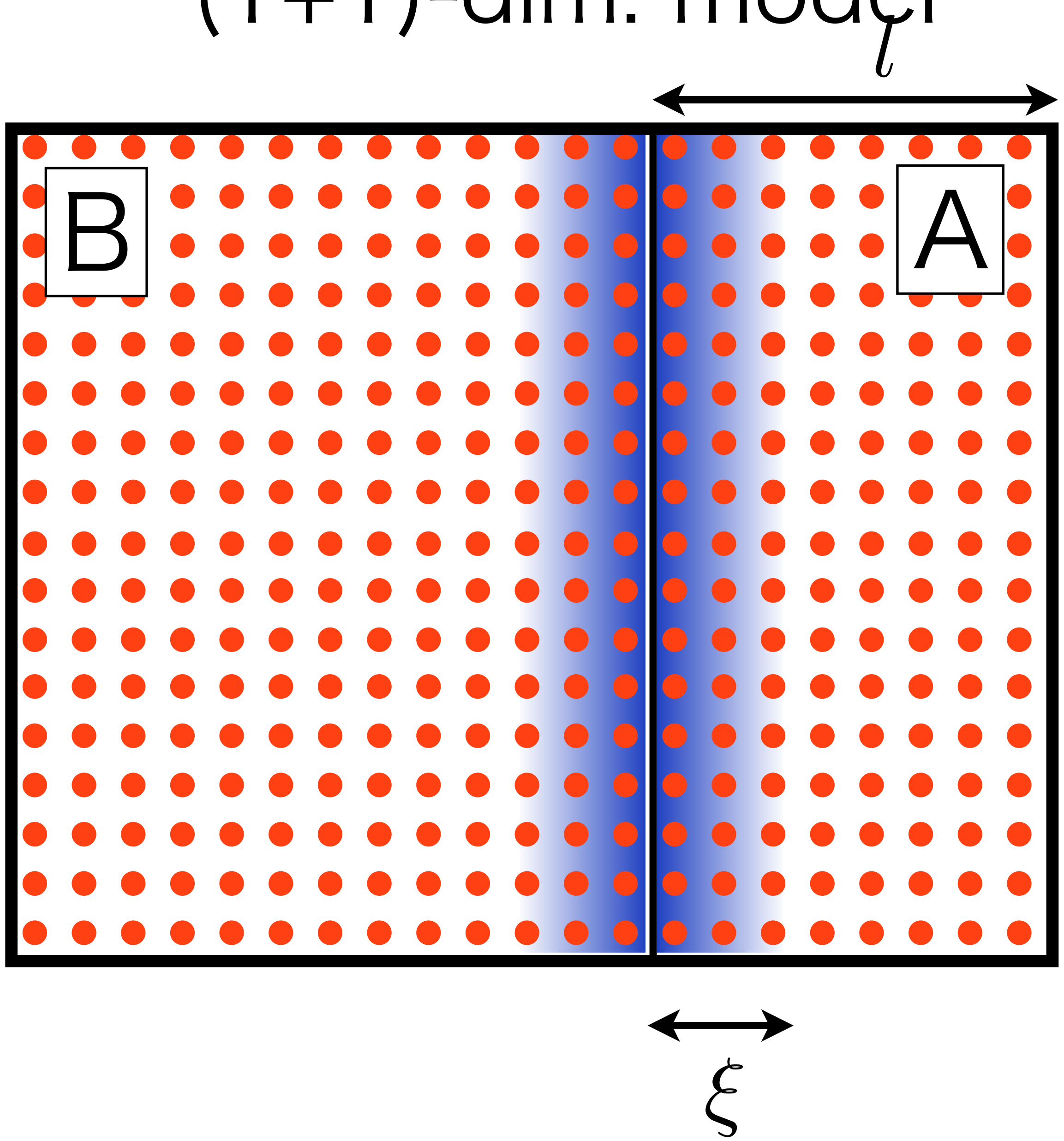
In the non critical system,

$l \ll \xi$

$$S_A(l) \sim \frac{c}{3} \log \frac{l}{a}$$

Schematic picture of E.E.

(1+1)-dim. model



ξ: correlation length of the system

In the non critical case,

$l \ll \xi$

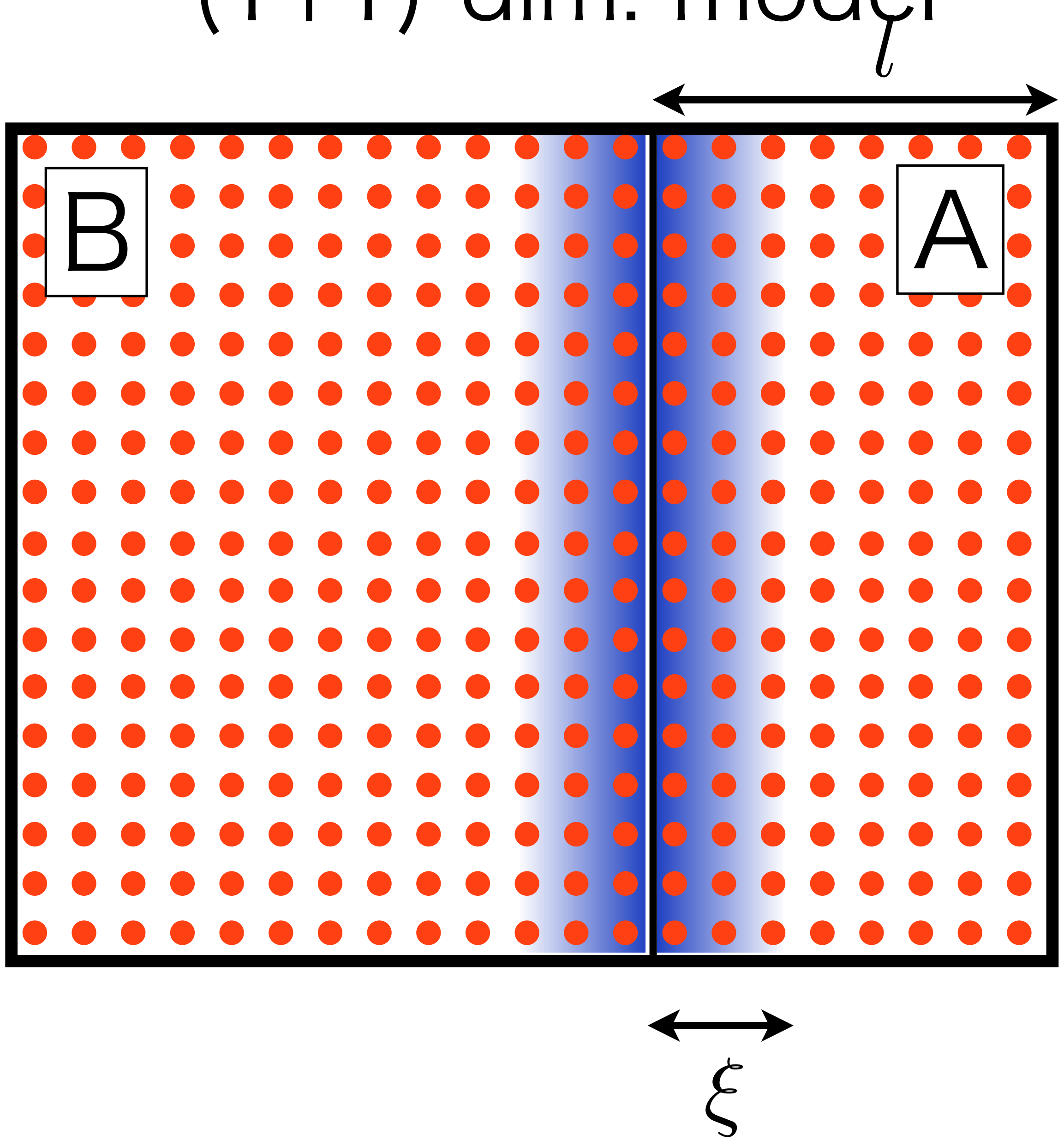
$$S_A(l) \sim \frac{c}{3} \log \frac{l}{a}$$

$l \gg \xi$

$$S_A(l) \to \frac{c}{3} \log \frac{\xi}{a}$$

Schematic picture of E.E.

(1+1)-dim. model



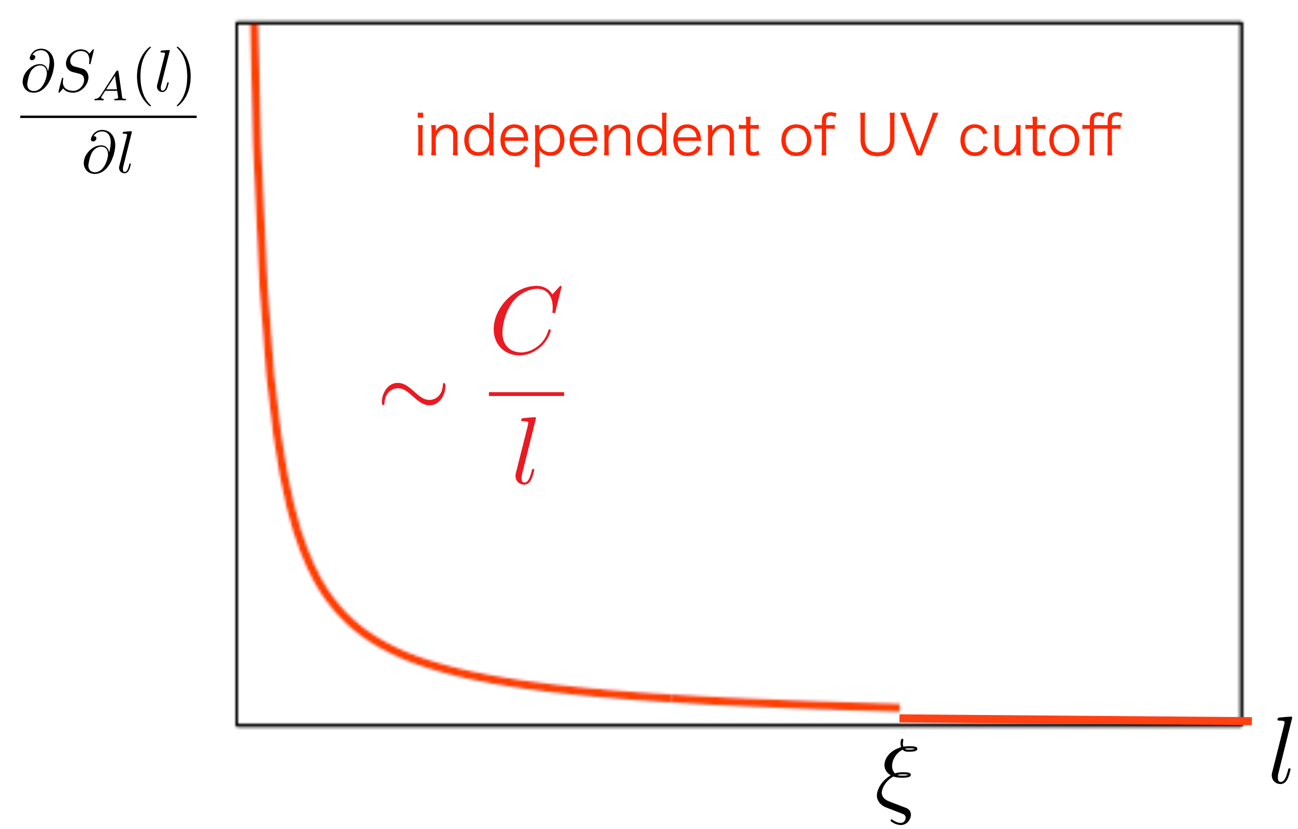
ξ: correlation length of the system

At the critical point or $l \ll \xi$ in the noncritical system

$$S_A(l) = \frac{c}{3}\log\frac{l}{a} + c_1$$

In the non-critical system,

$$S_A(l) \underset{l \gg \xi}{\longrightarrow} \frac{c}{3}\log\frac{\xi}{a}$$



Difficulties to obtain E.E. in 4d gauge theory

- UV cutoff dependence of 4d E.E.

2d

$$S_A(l) = c_0 \log(l/a)$$

4d

$$S_A(l) = c_0 \frac{\text{Area}}{a^2} - c_0' \frac{\text{Area}}{l^2} + c_1 \log(l/a) + (\text{regular terms})$$

Entropic C-function $$C(l) = l^3 \frac{1}{|\partial A|} \frac{dS}{dl}$$

Ryu and Takayanagi:PRL96(2006)181602
JHEP 0608(2006)045

cf) Holographic approach C is obtained by AdS and QFT

- (local) gauge invariance

E.E. for gauge theory

- P.V.Buividovich and M.I.Polikarpov PLB670(2008)141

  extended Hilbert space

- H.Casini, M.Muerta and J.A.Rosabal arXiv:1312.1183

  electric b.c.(electric center), magnetic center, trivial center

- D.Radicevic arXiv:1404.1391

  magnetic center

- W.Donnelly PRD85 (2012) 085004

  extended lattice construction

- S.Ghosh, R.M.Soni,S.P.Trivedi arXiv:1501.02593
- S.Aoki, T.Iritani, M.Nozaki et.al. arXiv:1502.04267

  maximally gauge invariant reduced density matrix

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red definitions are inadequate for E.E. or rhoA

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  Our work

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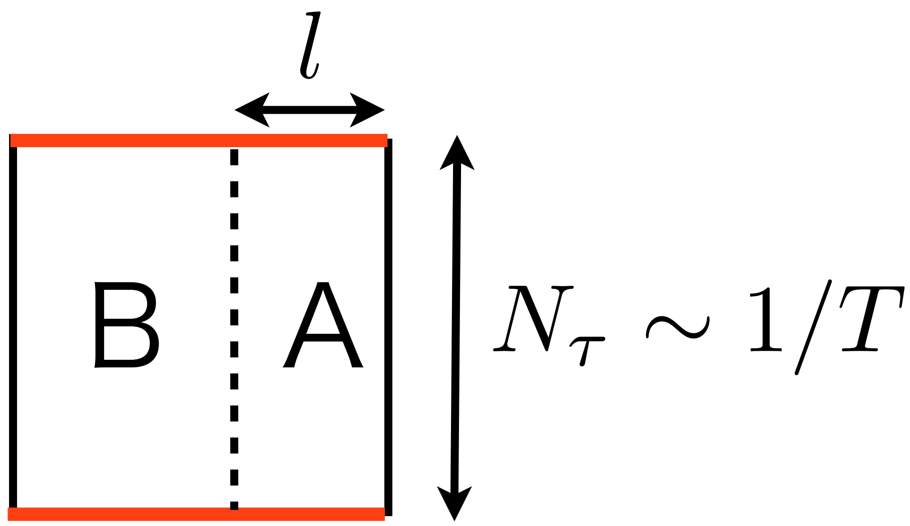
Replica method

Calabrese and Cardy: J.S.M.0406(2004)P06002

Replica method

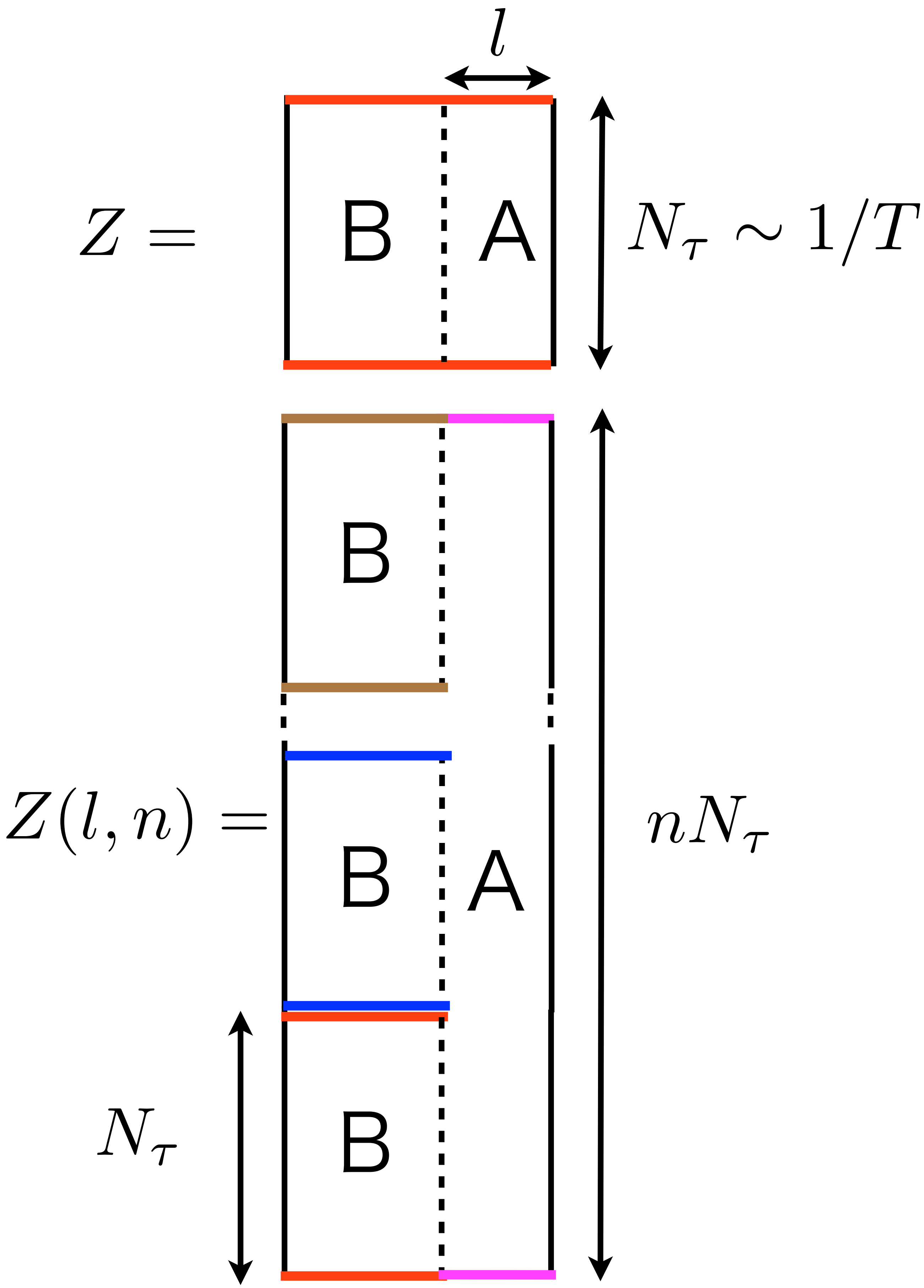
$Z =$ 

entanglement entropy:

$$S_A = -\lim_{n\to 1}\frac{\partial}{\partial n}\ln \mathrm{Tr}_A \rho_A^n$$

$$S_A(l) = -\lim_{n\to 1}\frac{\partial}{\partial n}\ln\left(\frac{Z(l,n)}{Z^n}\right)$$

Replica method



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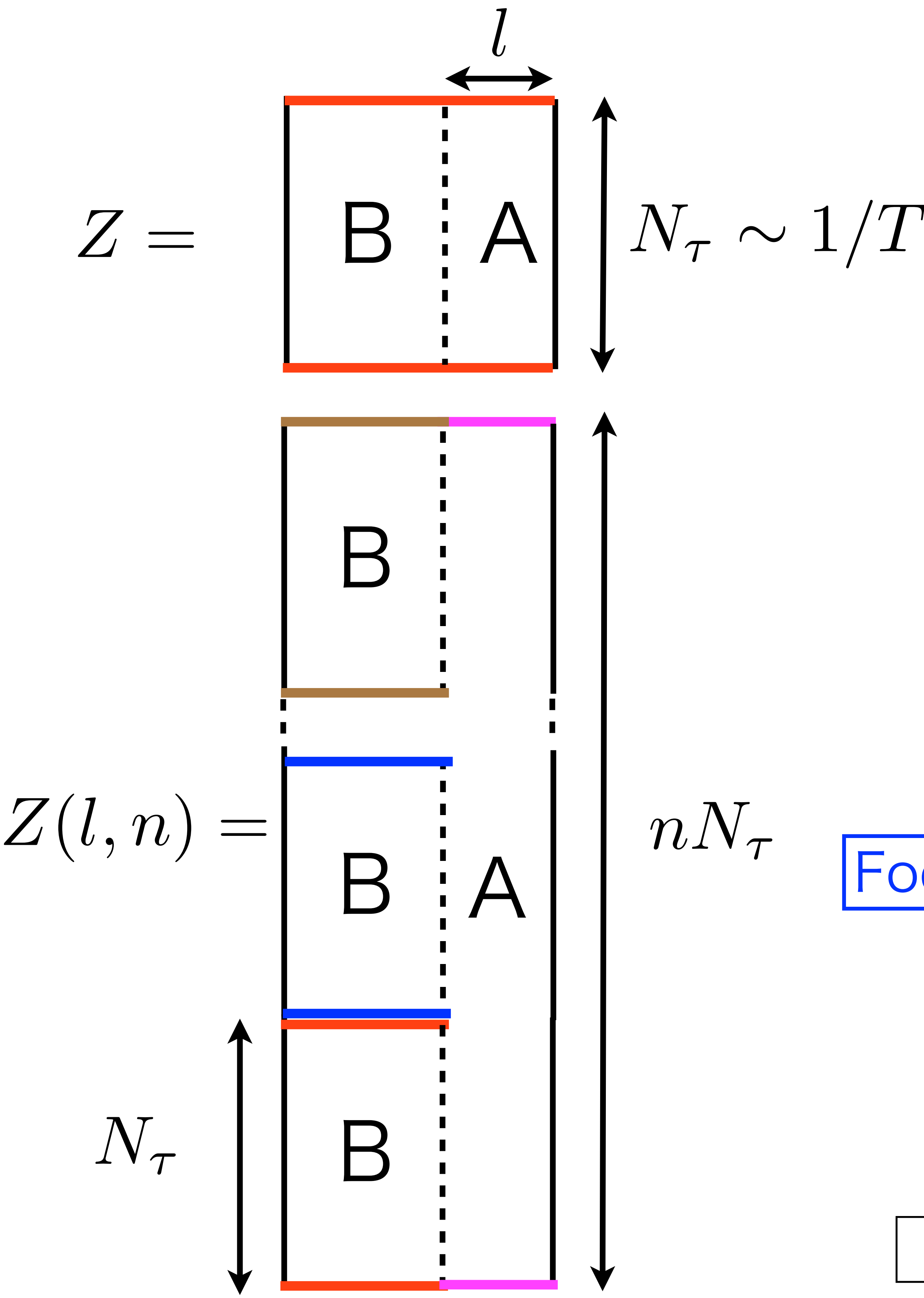
Replica method



entanglement entropy:

$$S_A = -\lim_{n\to 1} \frac{\partial}{\partial n} \ln \mathrm{Tr}_A \rho_A^n$$

$$S_A(l) = -\lim_{n\to 1} \frac{\partial}{\partial n} \ln \left(\frac{Z(l,n)}{Z^n} \right)$$

observable:

$$\frac{\partial S_A(l)}{\partial l} = \lim_{n\to 1} \frac{\partial}{\partial l}\frac{\partial}{\partial n} F[l,n]$$:free energy

$$\to \frac{F[l+a, n=2] - F[l, n=2]}{a}$$

Fodor (2009)

$$= \int_0^1 d\alpha \langle S_{l+a}[U] - S_l[U] \rangle_\alpha$$

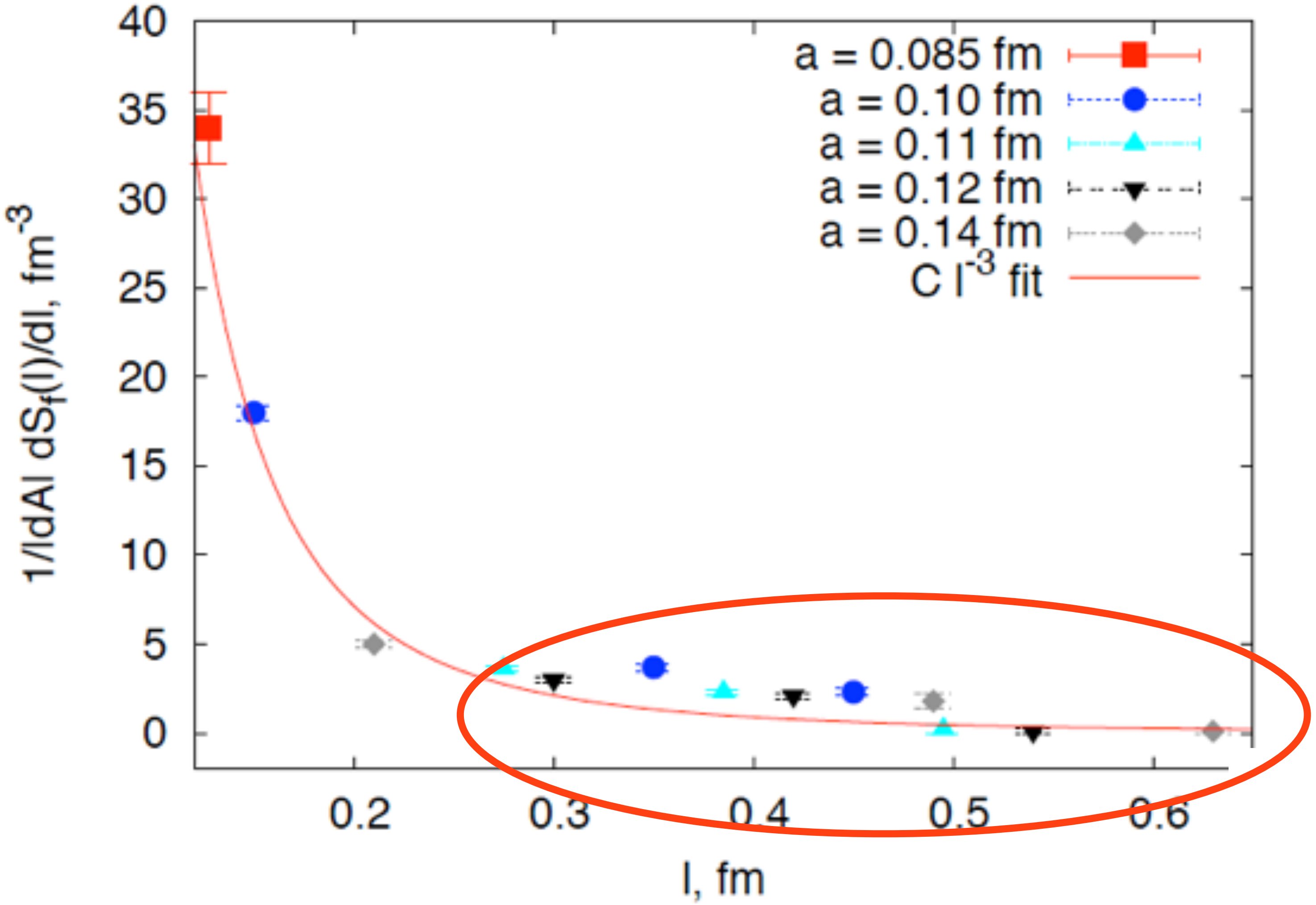
using the interpolation action

$$S_{int} = (1-\alpha) S_l[U] + \alpha S_{l+a}[U]$$

we measure the diff. of the action density

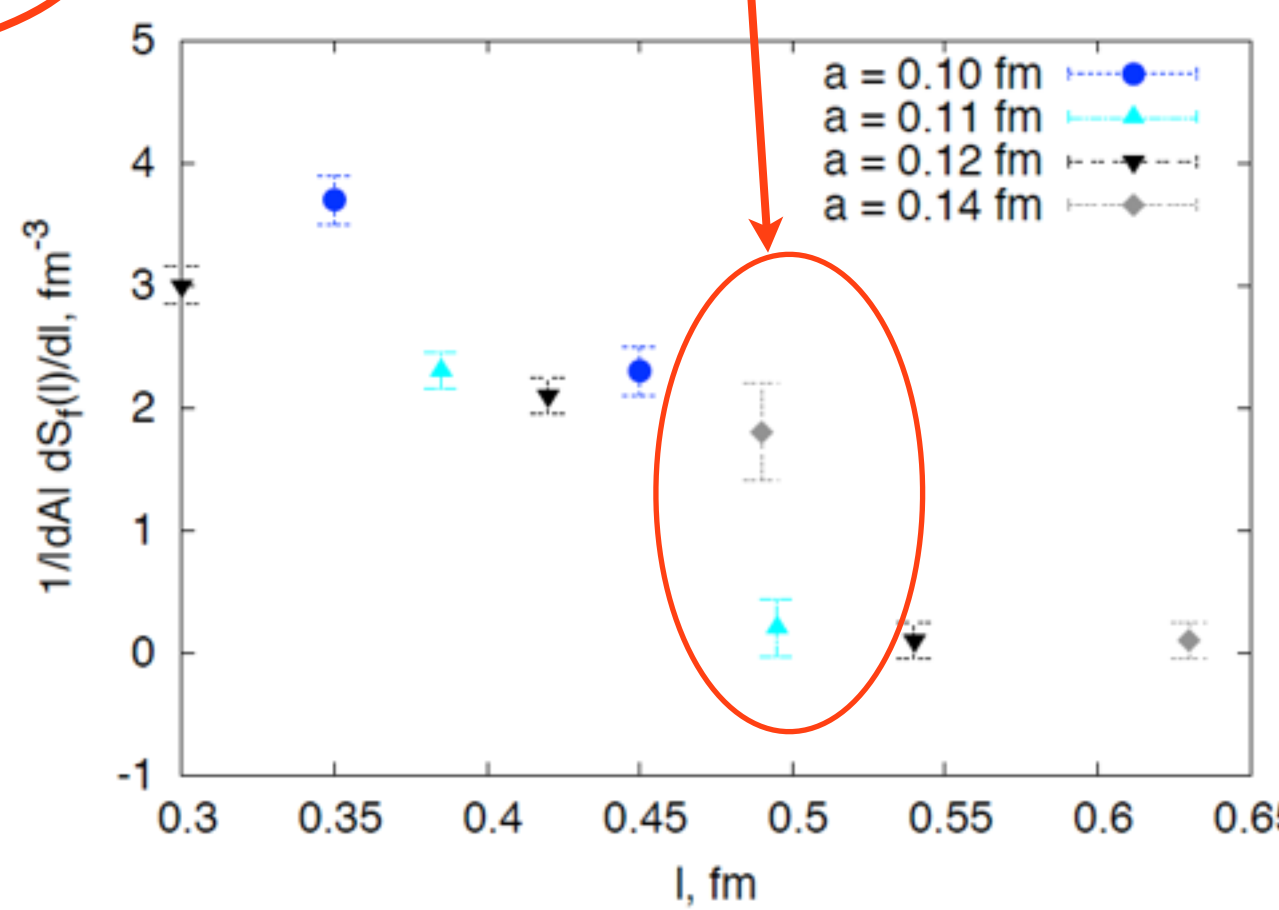
Simulation results

Lattice results for quenched SU(2)



Buividovich and Polikarpov: NPB802(2008)458

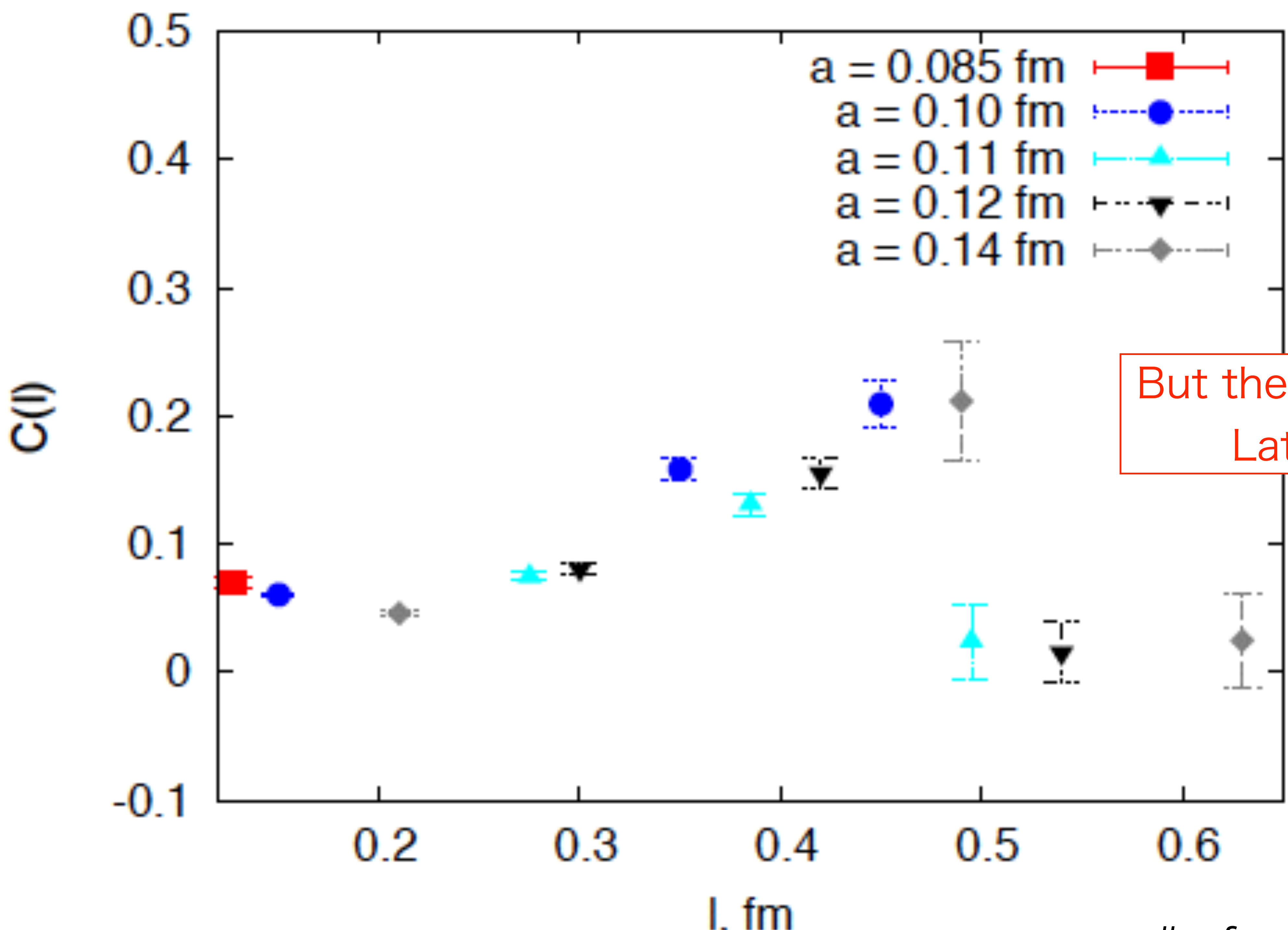
- in short range, 1/l^3 scaling
- discontinuity is clear!?



Entropic C-function

$$C(l) = l^3 \frac{1}{|\partial A|} \frac{dS}{dl}$$

should be constant in short l region



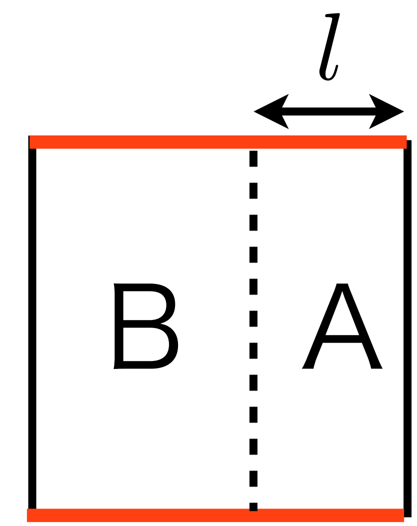
But the data is enhanced
Lattice artifact?

# of configs. is 18 -141

Our result

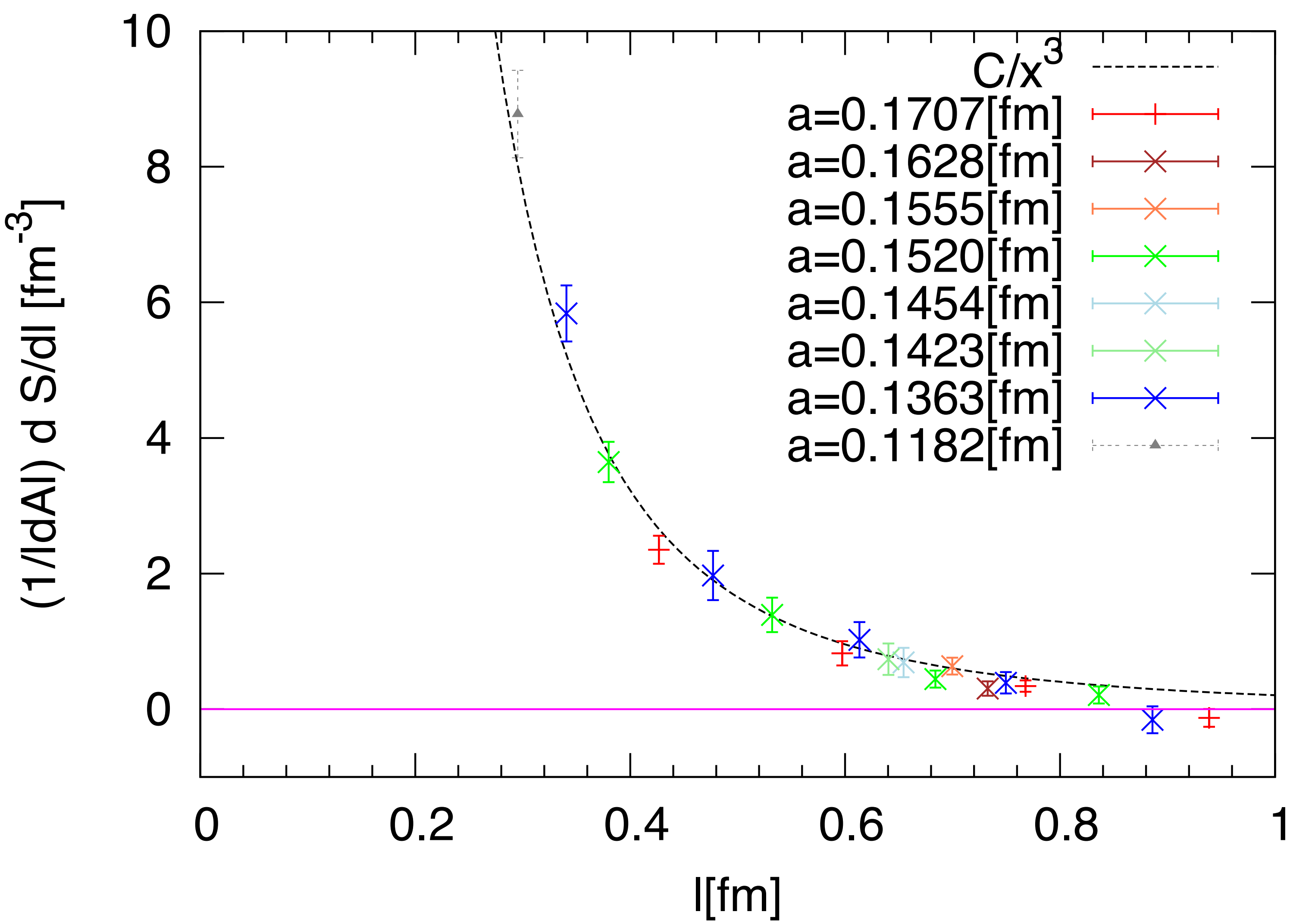
Simulation setup

- Wilson plaquette gauge action
- Ns=Nt=16, 32
- l/a=2,3,4,5,(6)
- beta=5.70 - 5.87
- # of configuration 12,000~84,000
- scale setting r_0 =0.5 fm and ALPHA coll.



Lattice results for quenched SU(3)

T=0, quenched QCD



We measure ~84,000 configs.

- in short range, 1/l^3 scaling
- the coefficient is C=0.2064(73)

cf.) C~0.09 in SU(2)

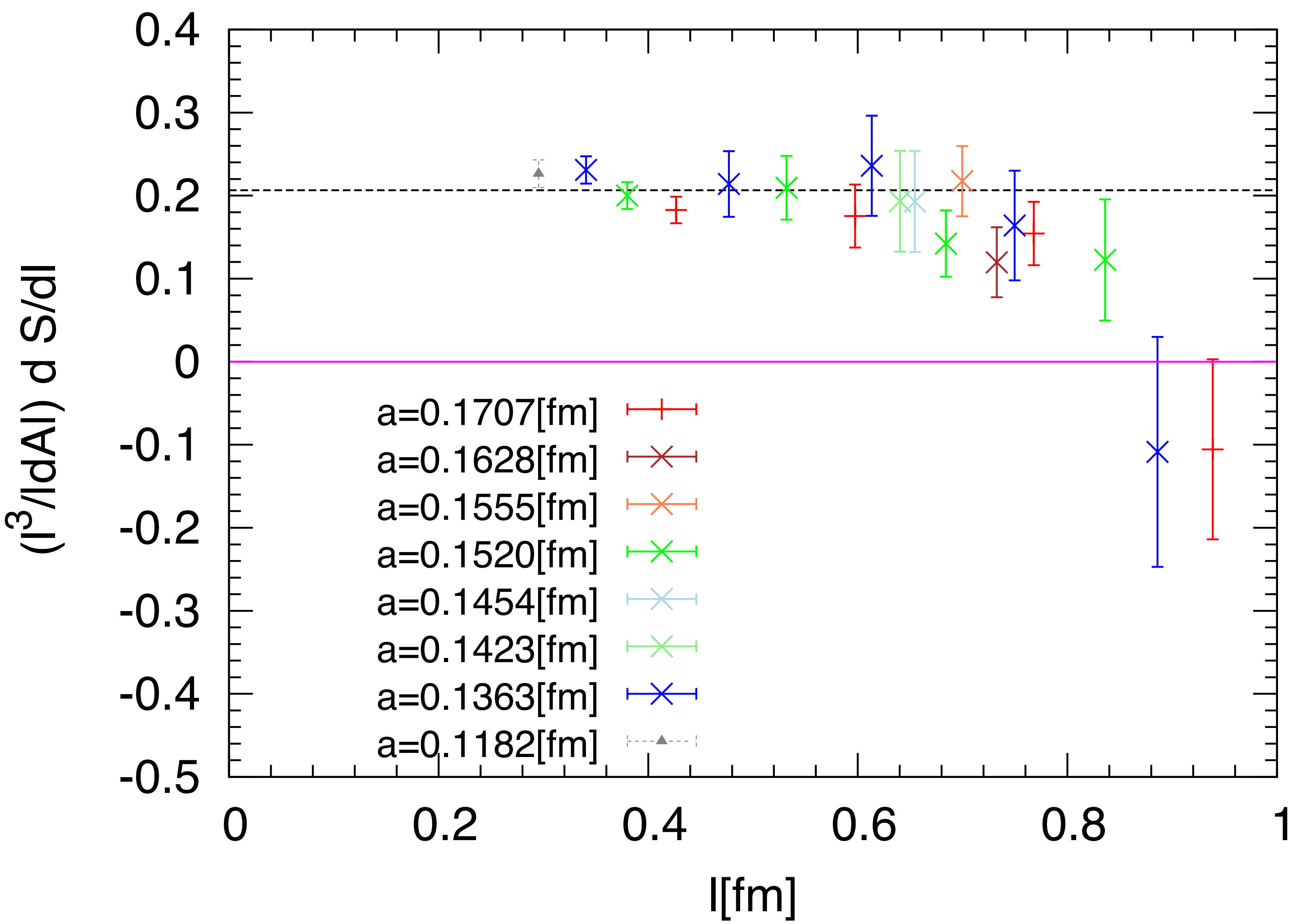
The Nc dependence is

$$\frac{0.2064}{0.09} = 2.29 \sim \frac{3^2}{2^2}$$

Entropic C-function

independent of UV cutoff

$$C(l) = l^3 \frac{1}{|\partial A|} \frac{dS}{dl}$$



In short l region, C is constant.

The discontinuity is not clear.

cf.) $\frac{1}{\Lambda_{QCD}} \sim 0.7[\mathrm{fm}]$

Comparison with Ryu-Takayanagi results

Ryu and Takayanagi:PRL96(2006)181602
JHEP 0608(2006)045

Holographic (or field theoretical) approach

(3+1)-dim. CFT $$\frac{1}{|\partial A|} S_A(l) = c\frac{N_c^2}{a^2} - c'\frac{N_c^2}{l^2}$$

c' is obtained by AdS and QFT

$c' \sim 0.0049$ for free real scalar theory

Estimation for non-abelian gauge theory

$$A_\mu^a \quad \begin{cases} a = 1, \cdots, 8 \\ \mu = 1, \cdots, 4 \end{cases}$$

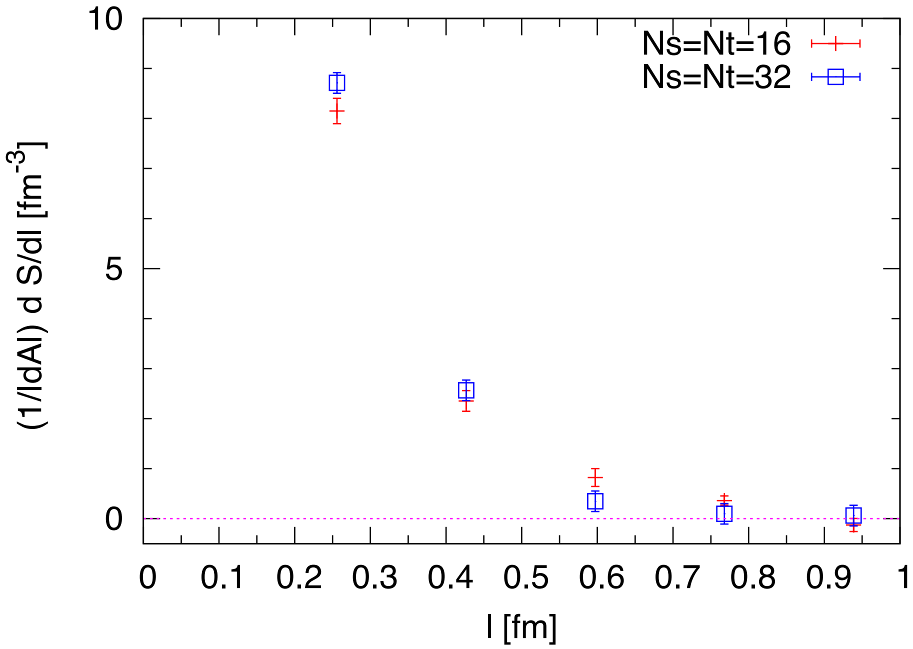
$$C_{\rm gauge} \sim 2c' \cdot 2 \cdot 8 \sim 0.1568$$

Our numerical result

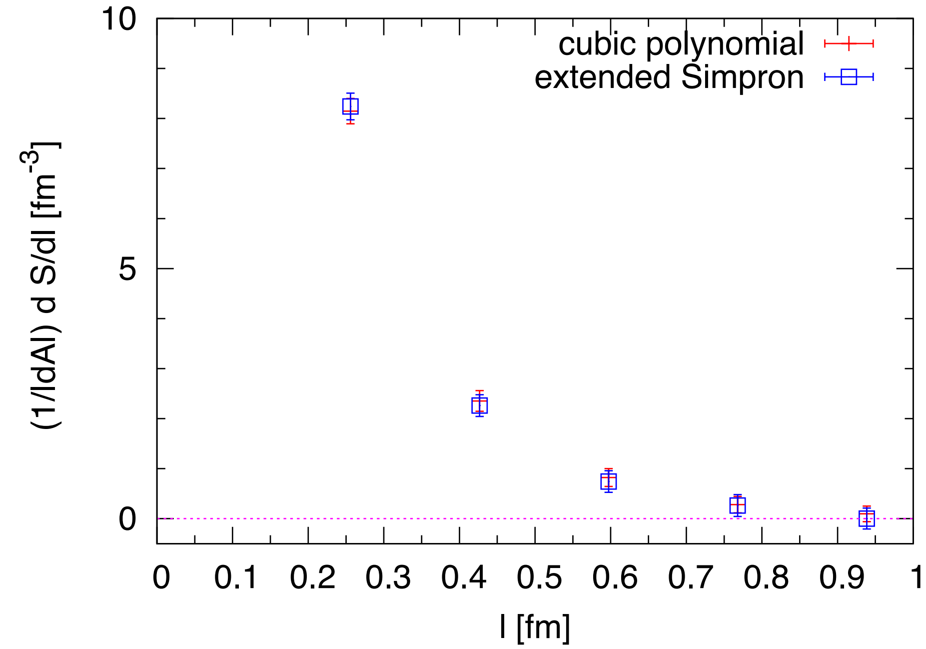
$$C_{\rm gauge} \sim 0.2064$$

Detail analyses

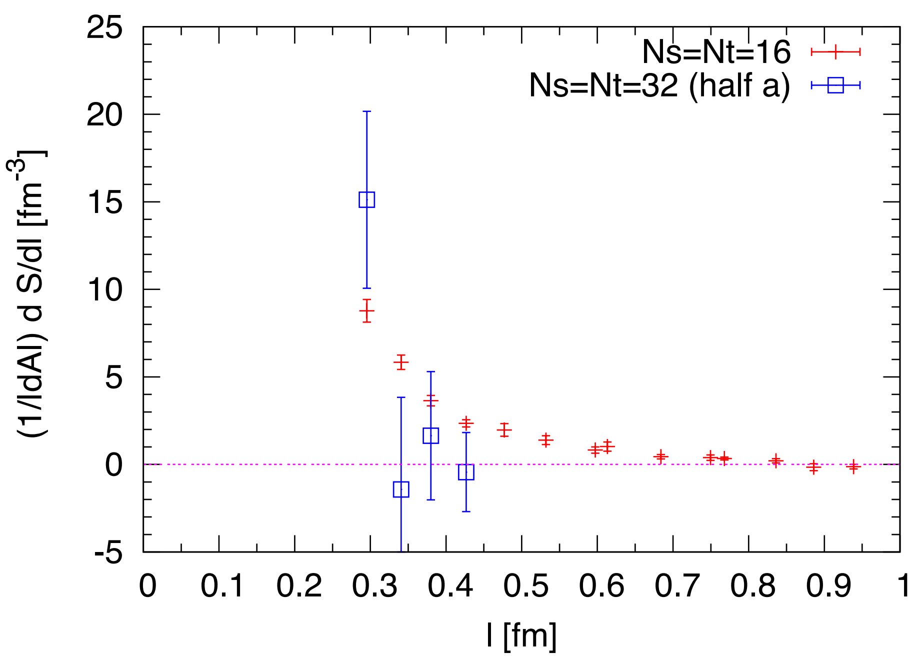
finite vol. effect



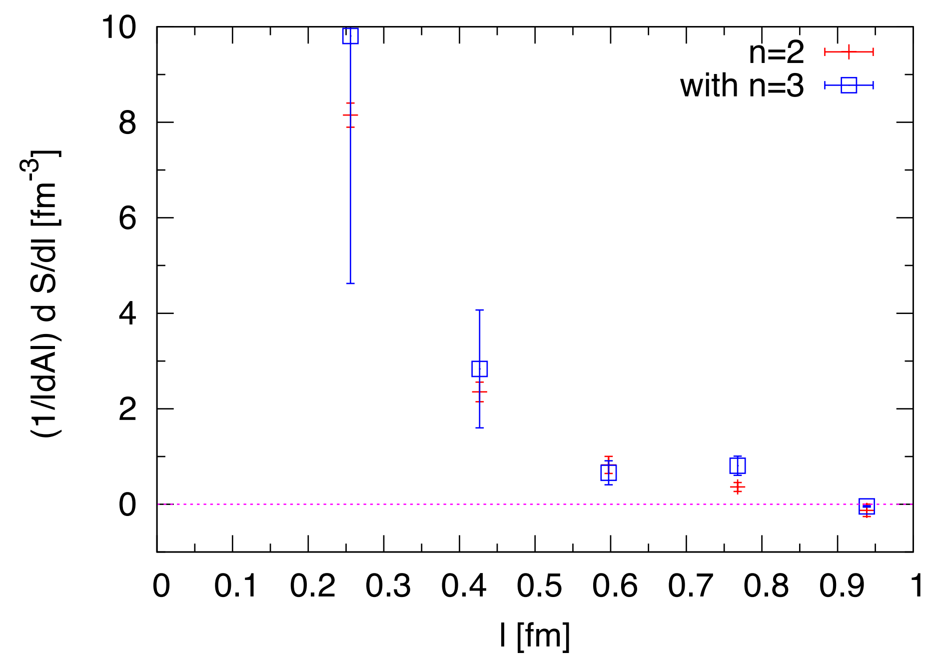
algorithm dependence of the numerical integration



UV cutoff dependence



replica number dependence



Summary

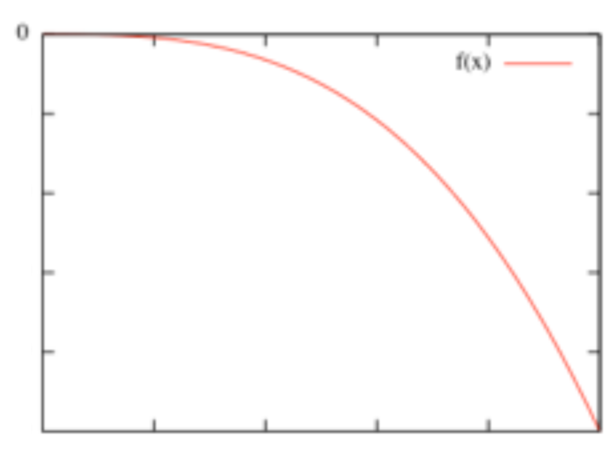
- This is the first precise determination of E.E. for quenched QCD
- Nc dependence in the short l region is Nc^2 as expected by AdS/CFT and field theoretical insights
- No discontinuity exists as contrast with SU(2) results
- Entropic C-function shows UV cutoff independence
- Value of C-function agrees with Ryu-Takayanagi work
- replica number (n ->1) dependence

Future directions for E.E. using the lattice

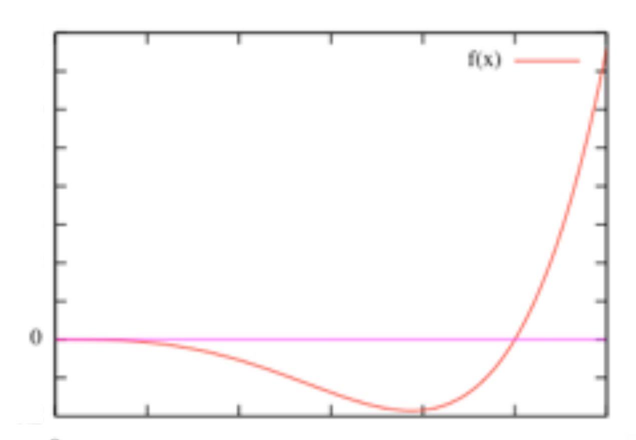
- QCD at zero T
  - give a novel observation for confinement
  - even in full QCD case
- QCD at finite T
  - gives the thermal entropy and the correlation length in QGP phase
- conformal window in 4dim Nf flavor QCD
  - would give the a-function and central charge

beta fn.

Nf<9



9<Nf<17



nontrivial IR fixed point is found by lattice simulation cf) E.I. PTEP(2013)083B01