Application of AdS/CFT Correspondence to Non-equilibrium Physics

Analog Black Holes and Non-equilibrium Steady States

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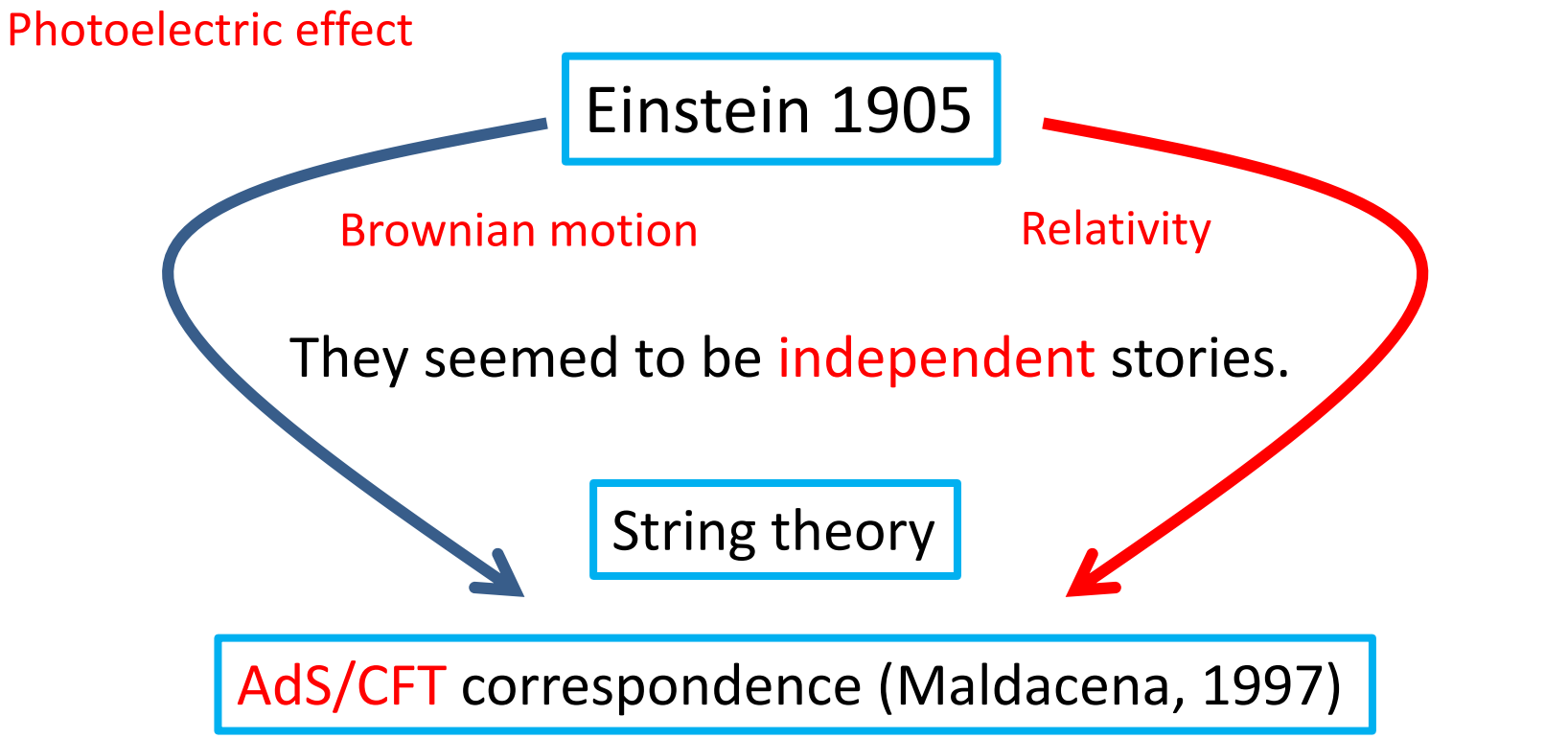
Refs.　H. Hoshino and S. N. in preparation.
S. N. and H. Ooguri　PRD88 (2013) 126003.
H. Hoshino and S.N.　PRD91 (2015) 026009.

We employ the natural unit: $k_B=c=\hbar=1$.

Physics in last century



100 years later, they turned out to be related:
they are just two sides of the same coin.

Fundamental question

in statistical physics

What is temperature?

We have many answers.

Definitions of temperature

$$P \propto e^{-E/T},\ t_E \approx t_E + 1/T$$ Statistical distributions

$$dE = TdS$$ Thermodynamics

$$D = T\mu$$ Fluctuation-dissipation relation

Diffusion const.

Two sides of the same coin.

AdS/CFT

We have another definition of temperature:

$$\xi^a \nabla_a \xi^b \Big|_{\text{Horizon}} = 2\pi T \xi^b \Big|_{\text{Horizon}}$$

Killing vector

Hawking temperature

Photo taken from http://www.iqs.com/iqs-blog/common-ground-quality-management-risk-management/

One side of the coin: general relativity

Einstein has formulated the theory of gravity in terms of geometry of space and time: general relativity.

"Energy-momentum deforms the spacetime."

Gravitational attraction can be explained as an effect of curved spacetime.

Einstein-Hilbert action:

Curvature:~combination of second derivatives of the metric

$$S = \frac{c^3}{16\pi G_N}\int d^{d+1}x\sqrt{-g}\left(R - 2\Lambda\right) + S_{\text{star}}$$

Cosmological constant

Metric: defines unit length in the geometry

Einstein's equation:

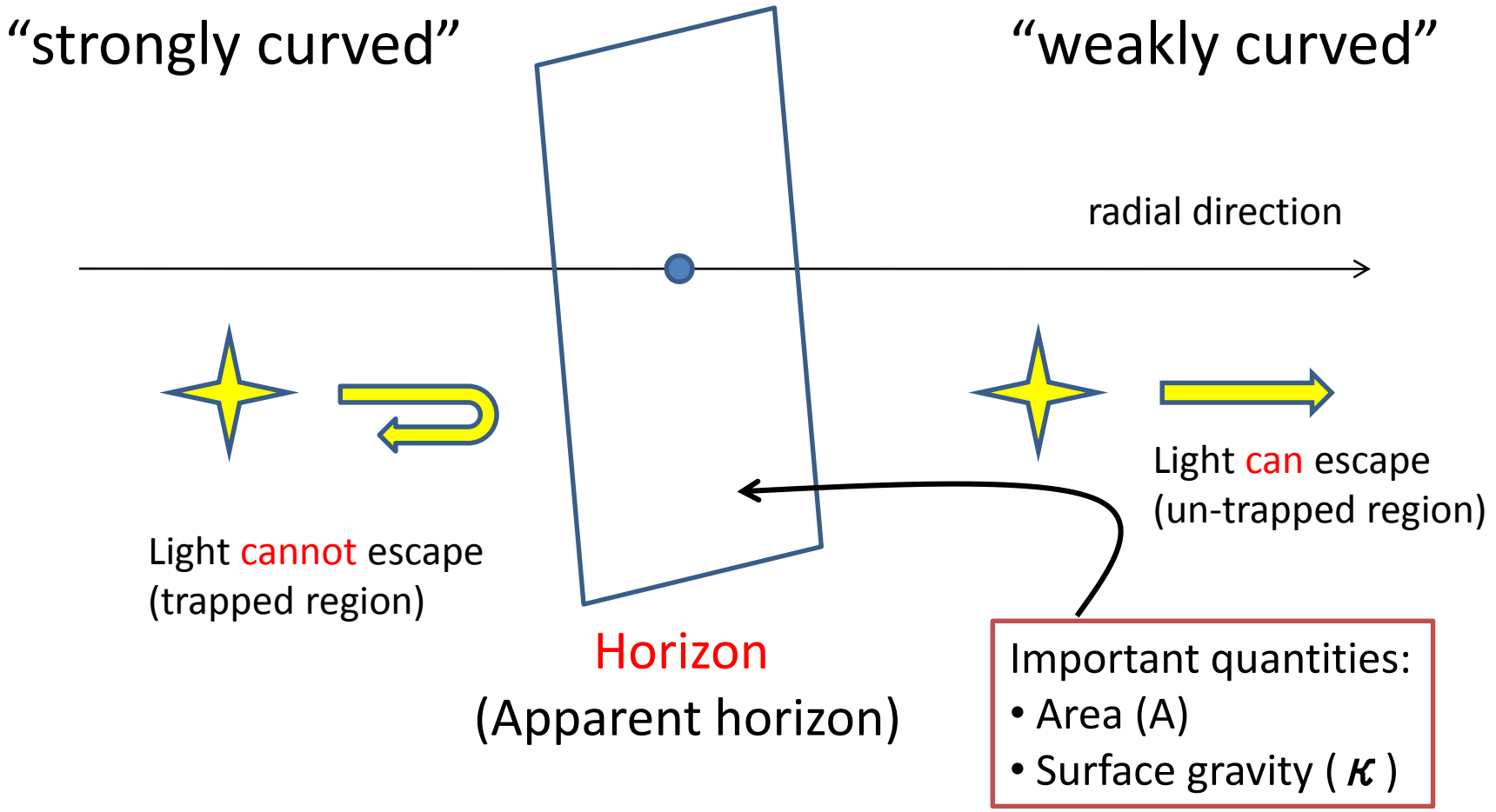
$$R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G_N}{c^4}T_{\mu\nu}^{(\text{star})}$$

2nd-order differential equation

Picture:http://www.faculty.iu-bremen.de/course/fall02/c210101/students/BlackHoles/

Black hole

A solution to the Einstein's equation.



Black hole mimics thermodynamics

Black holes obey:

$$dM = T_H dS_{BH} \qquad S_{BH} = \frac{k_B c^3}{\hbar G_N} \frac{A}{4}$$

Area of the horizon

Mass of the black hole

Hawking temperature

Newton's constant

This resembles of the first-law of thermodynamics

$$dE = TdS$$

This is not only an analogy. A black hole radiates a black-body radiation with Hawking temperature: we can assign a temperature to a black hole.

Hawking, S. W. (1974). "Black hole explosions?". *Nature* **248** (5443): 30.

Temperature in black hole.

Black hole thermodynamics

Natural unit: $k_B=c=\hbar=1$.

	Thermodynamics	Black hole
0-th law	T= const. at the equilibrium.	κ is constant in the static solution.
1st law	dE=T dS	$dM=[\kappa/(8\pi G_N)]dA$
2nd law	Entropy never decreases.	The area of horizon (A) never decreases.
3rd law	We cannot reach T=0 in any physical process.	κ cannot reach zero in any physical process.

κ: surface gravity (the gravitational acceleration at the horizon of the black hole)

G_N: Newton's constant, M: mass of the black hole

A: area of the horizon

κ and A mimic T and S, respectively.

$$T=\frac{\kappa}{2\pi},\quad S=\frac{A}{4G_N}$$

$T_H=\kappa/2\pi$, by Hawking.

This does not seem to be just a coincidence.

An answer from string theory:

AdS/CFT correspondence

[Maldacena, 1997]

A quantum field theory of gauge particles

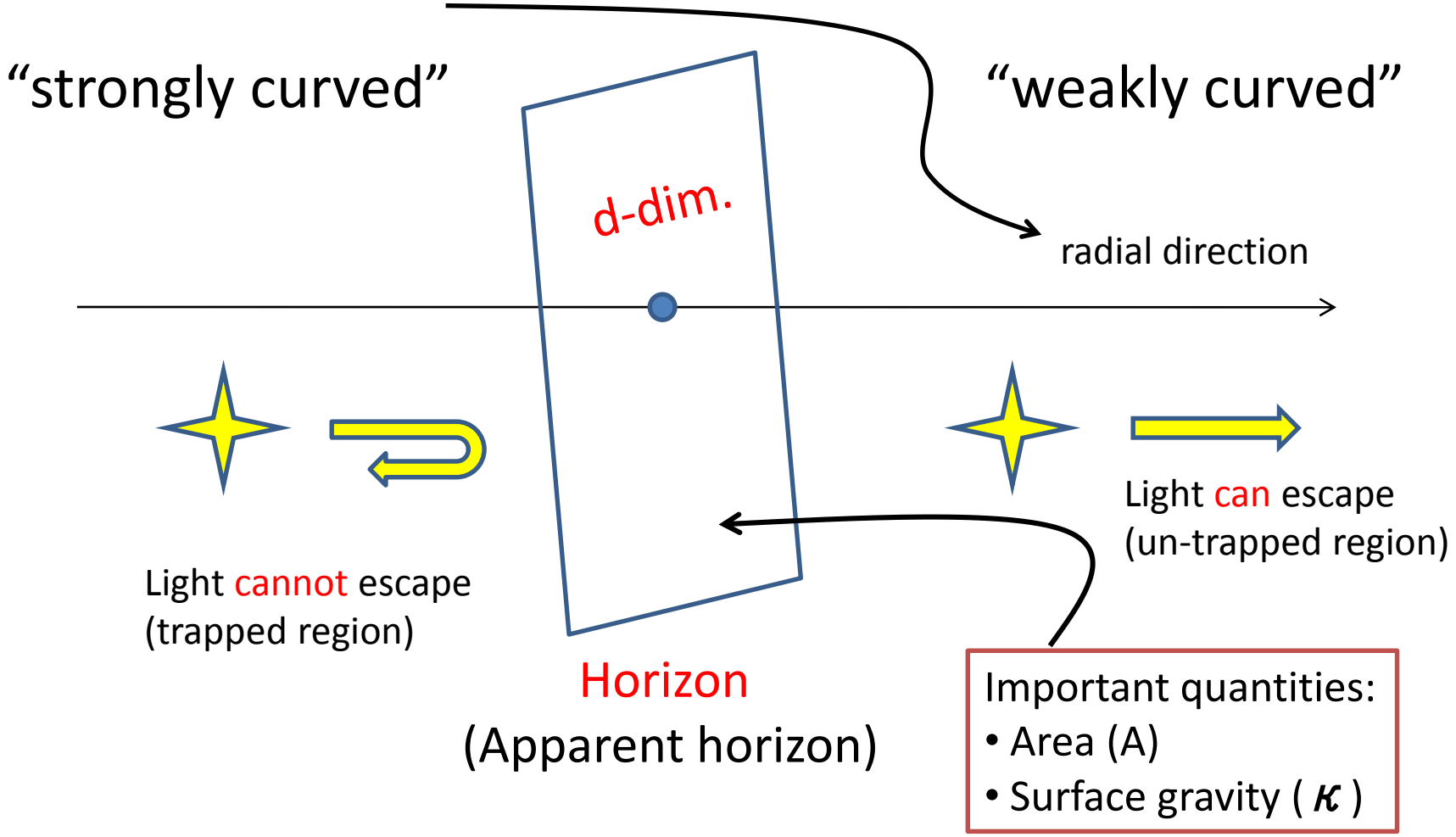
Equivalent

A classical theory of gravity on a curved geometry in higher-dimensions.

A conjecture, but no (established) contradiction has been found within more than 10,000 citations.

Higher-dimensional gravity?

If the BH corresponds to the thermodynamics of a d-dimensional system, we need the extra direction to define the horizon.



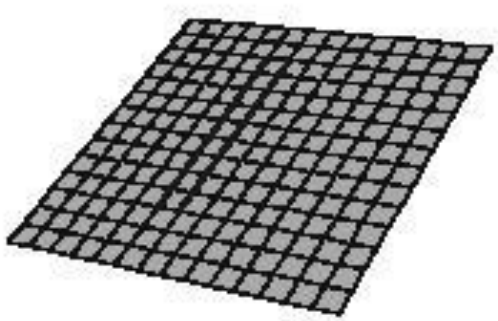
What is AdS?

AdS: Anti de Sitter spacetime

A spacetime whose scalar curvature is negative and constant.

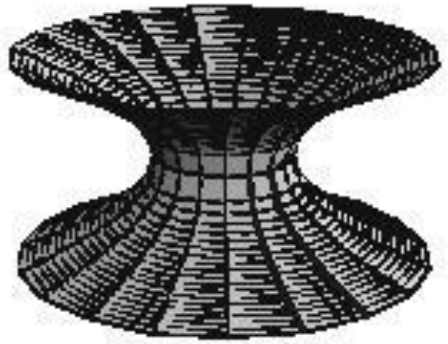
dS: de Sitter spacetime
(positive constant curvature)

It has a boundary.



Zero curvature (flat)

Positive curvature



Negative curvature

Picture:http://www.faculty.iu-bremen.de/course/fall02/c210101/students/BlackHoles/

An answer from string theory:

AdS/CFT correspondence [J. Maldacena, 1997]

A quantum field theory of gauge particles ⟺ A classical theory of gravity on a curved geometry (typically on AdS_5)

The correspondence itself is at the level of microscopic theory.

Finite temperature

Many-body system of gauge particles at temperature T ⟺ Black hole geometry (typically on AdS_5) at temperature T

[E. Witten, 1998]

Mystery in gravity

Many-body system of gauge particles at temperature T

Black hole geometry (typically on AdS_5) at temperature T

- AdS/CFT is a correspondence at the level of microscopic theory of gauge particles.
- What we have done in the gravity side was just solving the differential equation.

This is a solution to the Einstein's equation (2nd-order differential equation).

Who did the coarse graining to get the thermodynamics?

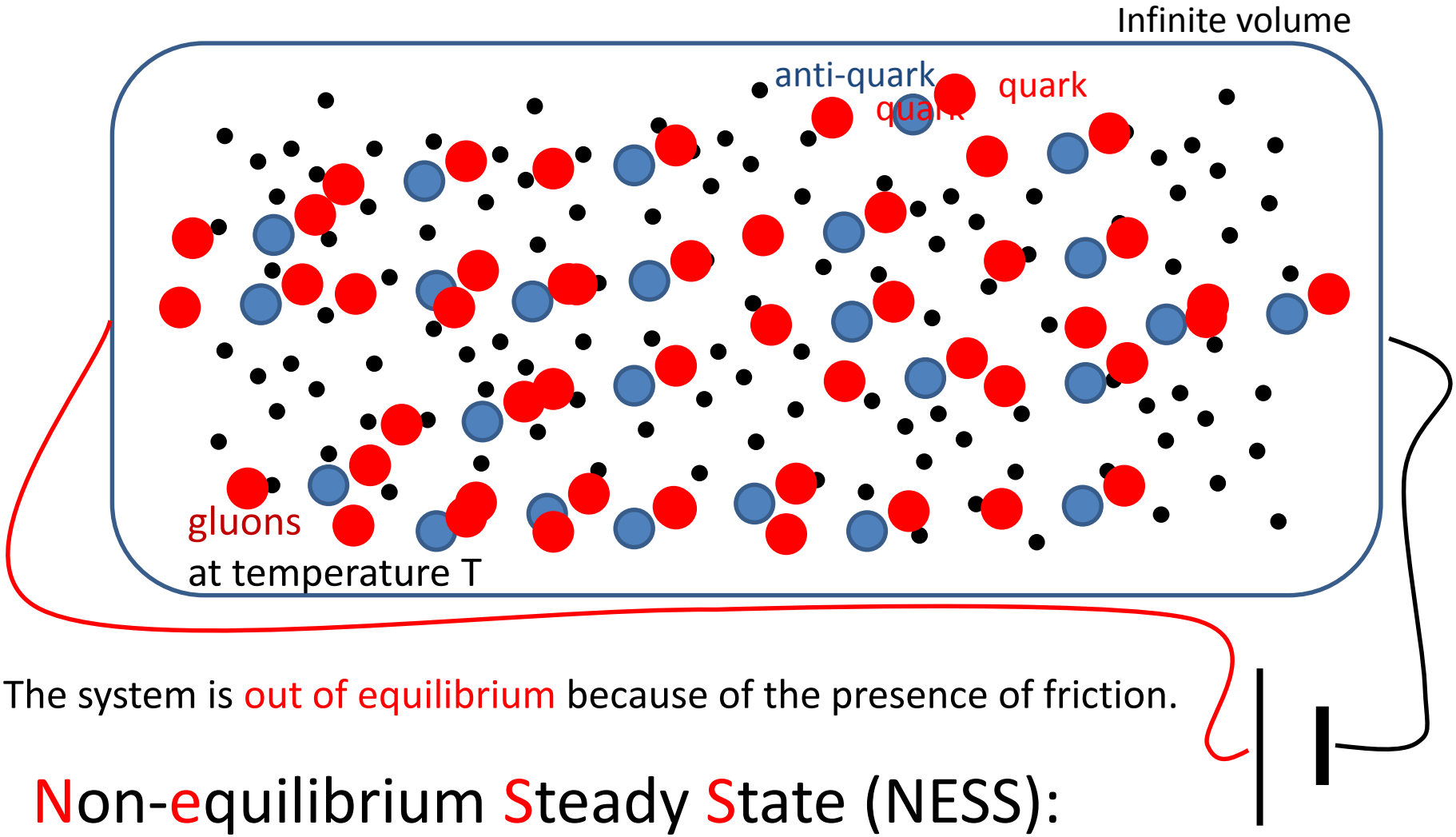
Who did the coarse graining?

An "observational fact" in mathematical physics is that, starting with the microscopic theory of the gauge theory, we can reach a description of macroscopic physics by using the AdS/CFT correspondence in terms of gravity.

Micro to macro without "explicit" coarse graining.

I will use this property of gravity to get further information on non-equilibrium physics in this talk.

The system we consider: a "conductor"



The system is out of equilibrium because of the presence of friction.

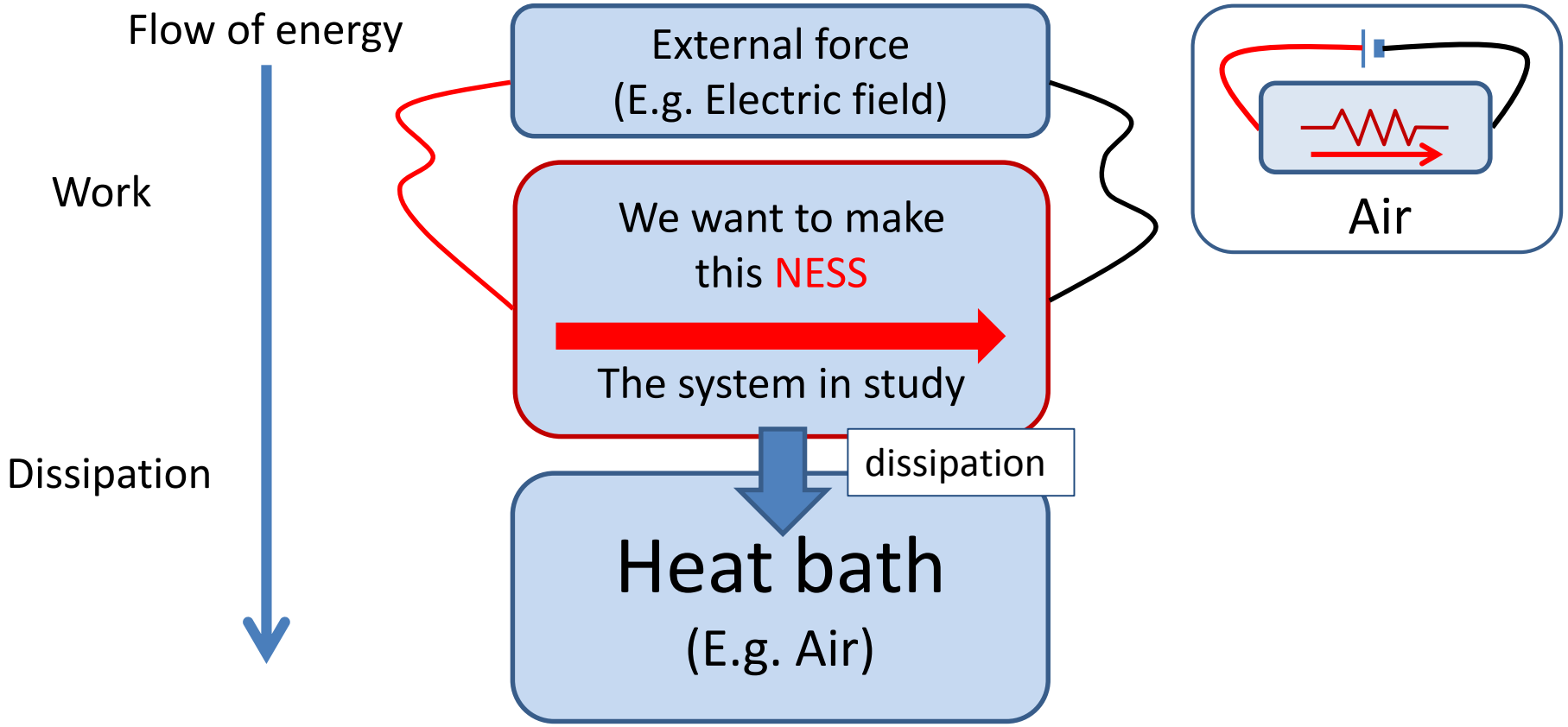
Non-equilibrium Steady State (NESS):

- The system in study is out of equilibrium.
- But time-independent (steady).

Setup for NESS

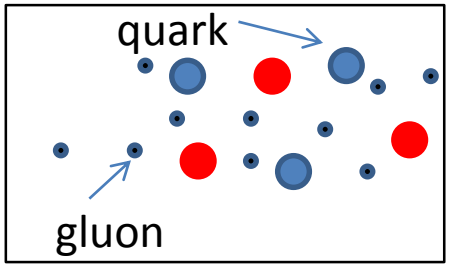
External force and heat bath are necessary.

Power supply drives the system our of equilibrium.



The subsystem can be NESS if the work of the source and the energy dissipated into the heat bath are in balance.

How to realize the steady state with constant current?



External force acting on the quark charge

quark sector (charge carrier)

current

N_c

dissipation

The system will be heated up.
However, the heat capacity of the gluon sector will be very large at N_c>>1.

Heat bath at constant T.

gluon sector (phonon/photon)

N_c^2-1

N_c>>1 (probe approx.)

We can consistently prepare NESS at the limit of N_c>>1 .

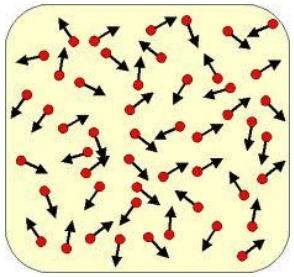
Map into the gravity dual

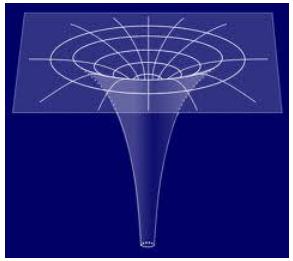
A strongly-interacting
quantum gauge theory

AdS/CFT

equivalent

A classical gravity
(general relativity)
on a curved spacetime
in higher dimensions.





Heat bath
(gluons)

Black Hole

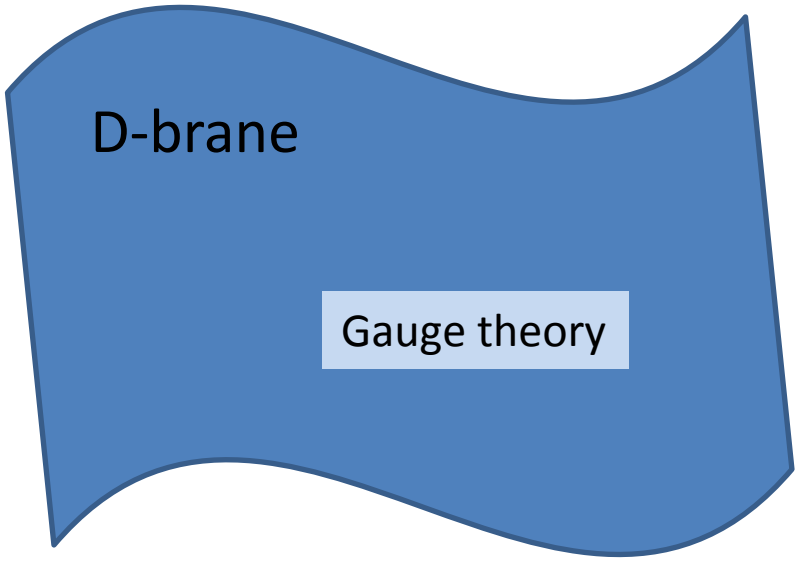
Charged particles
(quarks/antiquarks)
in the heat bath

An object that is
called D-brane.

Picture of many-body system is taken from internet.
Picture of black hole is taken from https://www.kahaku.go.jp/exhibitions/vm/resource/tenmon/space/theory/theory06.html

D-brane

- A solitonic object in superstring theory.
  It is like a membrane-like object.
  (Dp-brane: (p+1)-dimensional object)
- It effectively describes a system of gauge particles.



AdS/CFT based on D3-D7

SU(Nc) N=4 Supersymmetric Yang-Mills (SYM) theory at large-Nc with $\lambda = g_{YM}^2 Nc >> 1$.
（Quantum field theory） Finite T

+ quark sector (N=2 hyper-multiplets)

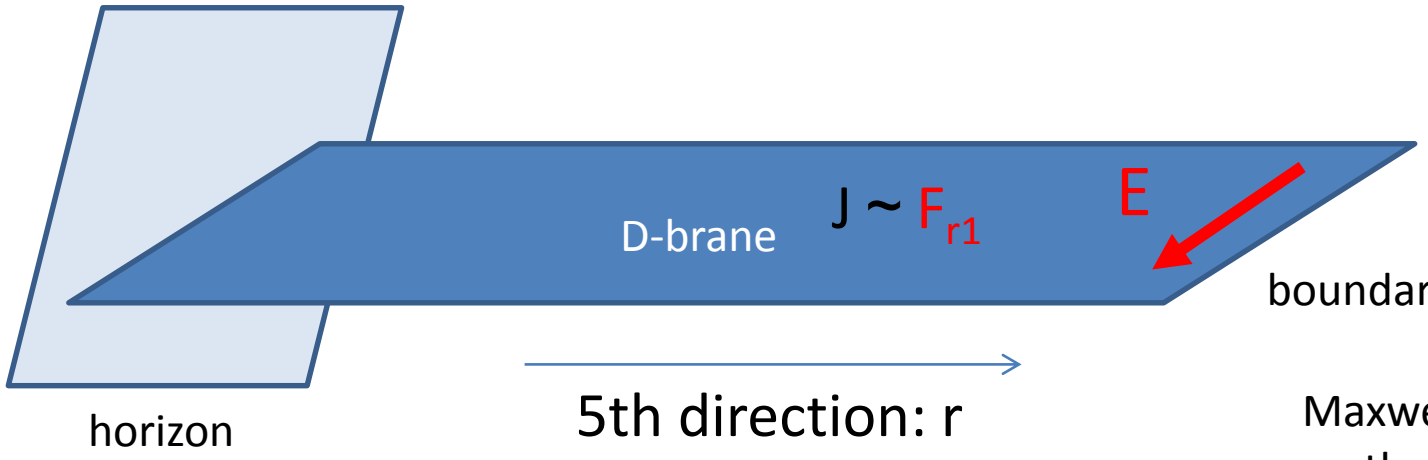
Equivalent

Type IIBsupergravity at the classical level on weakly curved AdS-BH $\times S^5$

+ D7-brane on this curved spacetime

A cartoon in the gravity dual

[Karch and O'Bannon, 2007]



Maxwell field
on the D-brane

$$L_{\mathrm{DBI}} = (\mathrm{tension})\sqrt{-\det\left(\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} + F_{ab}\right)}$$

We apply an external electric field E.

$$A_1 = -Et + h(r)$$

$$J = \frac{\partial L}{\partial F_{r1}}$$

Relationship between E and J

[Karch and O'Bannon, 2007]

$$\left(F_{r1}\right)^2 = J^2 \frac{g_{rr}}{\left|g_{tt}\right|} \frac{E^2 - \left|g_{tt}\right| g_{xx}}{J^2 - \left|g_{tt}\right| g_{xx}^2}$$

Again, we have a special point r_* given by

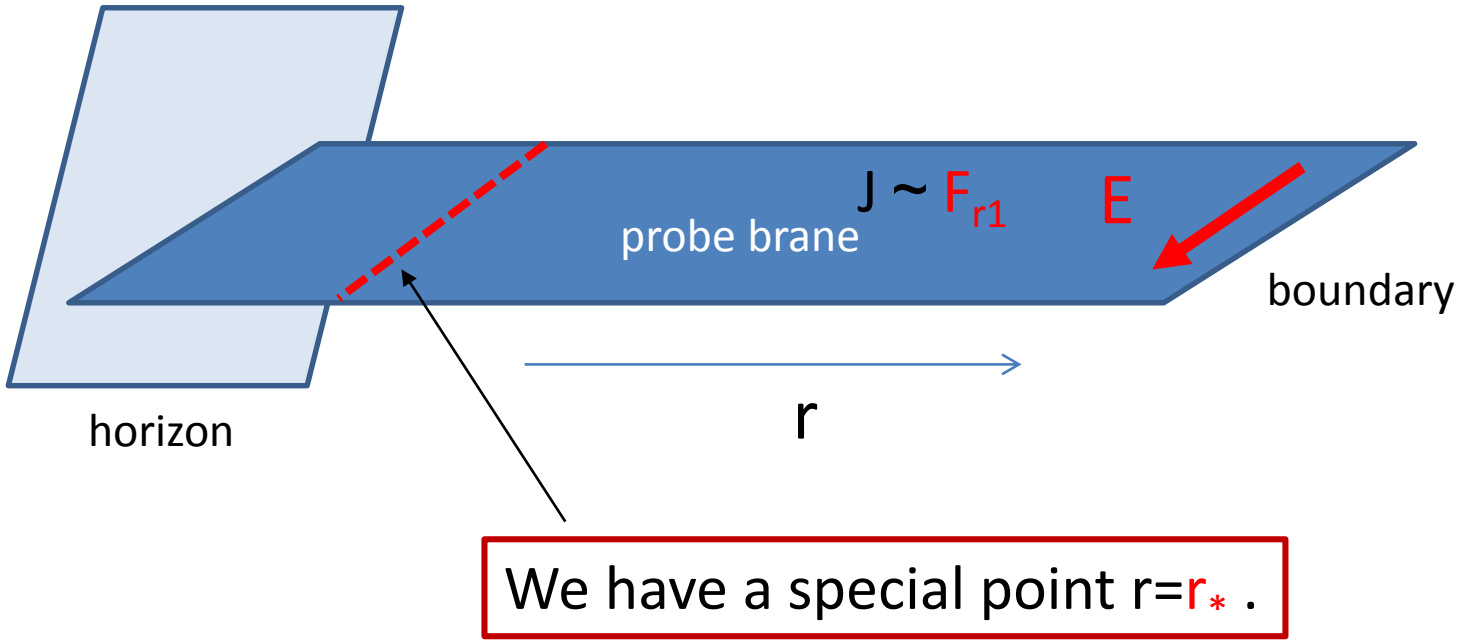
$$E^2 - \left|g_{tt}\right| g_{xx} = 0,$$

and J is given by $\left(J^2 - \left|g_{tt}\right| g_{xx}^2\right)\Big|_{r_*} = 0$ in terms of E.

J(E) is obtained as a non-linear function.

Special point on the D-brane

[Karch and O'Bannon, 2007]



We have a special point $r=r_*$.

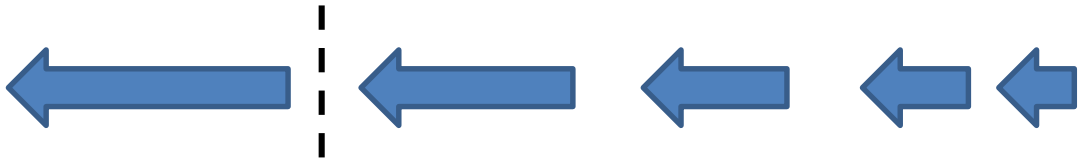
This plays a role of another horizon seen by the fluctuations (of current, for example) in NESS.

An analog black hole

[Gubser 2008, Kim-Shock-Tarrio 2011, Sonner-Green 2012]
[S.N. and H. Ooguri, 2013]

What is analog black hole?
"non-gravity black hole"

Sonic black hole in liquid helium.



Fast — Sonic horizon where the flow velocity exceeds the velocity of sound. — Slow

- The sound cannot escape from inside the "horizon".
- It is expected that the sonic horizon radiates a "Hawking radiation" of sound at the "Hawking temperature".

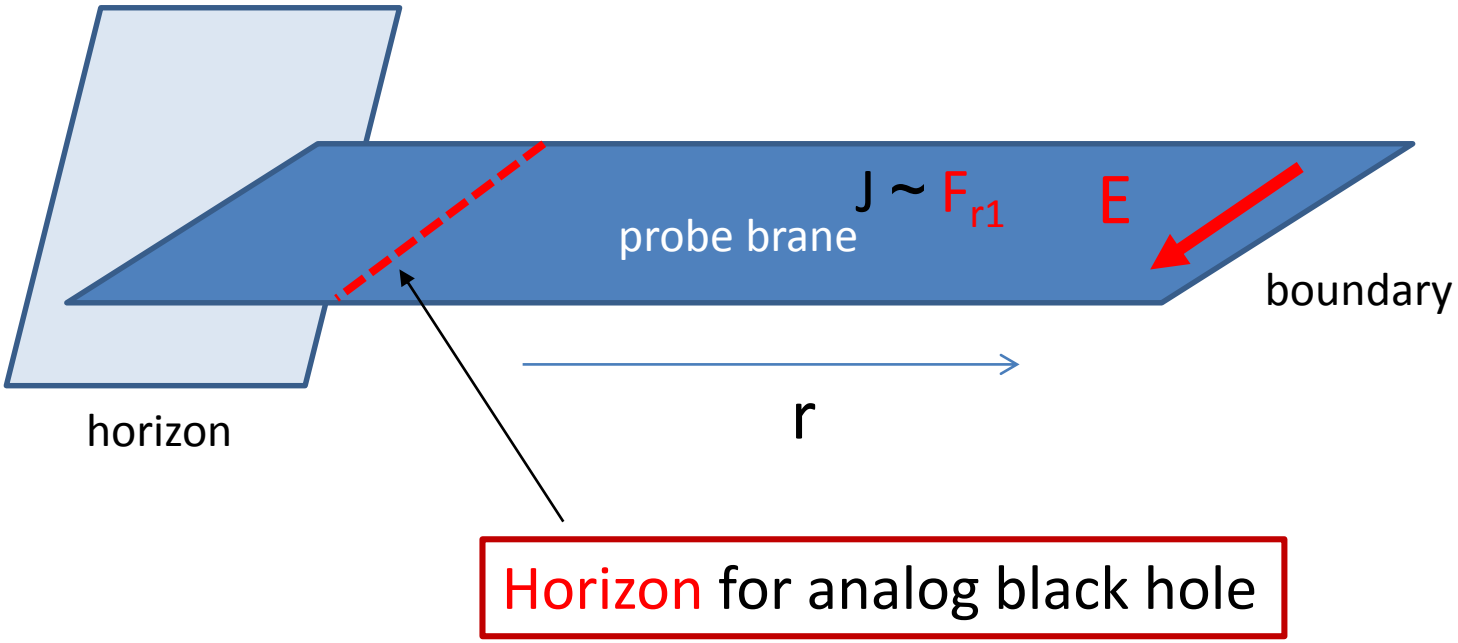
[W. G. Unrhu, PRL51(1981)1351]

We do have "temperature", but any "thermodynamics" with analog black hole has not been established so far.

[See for example, M. Visser, gr-qc/9712016]

Special point on the D-brane

[Karch and O'Bannon, 2007]



We do have another temperature in this sense.

[Gubser 2008, Kim-Shock-Tarrio 2011, Sonner-Green 2012]
[S.N. and H. Ooguri, 2013]

Now we have two temperatures

Black hole horizon
gives the temperature
of the heat bath.

The effective horizon on the D-brane
gives a different "Hawking temperature"
that governs the fluctuations in NESS.

We call this effective temperature T_{eff} of NESS.

If the system is driven to NESS,
$r_H < r_*$ at the order of E^2.

Two temperatures appear only in the non-linear regime.

The meaning of T_{eff}

Fluctuation of electro-magnetic fields on the D-brane ⟺ Fluctuation of current density

We can compute the correlation functions of fluctuations by using the technique of AdS/CFT.

$$\int dt \left\langle \delta J(t) \delta J(0) \right\rangle \Big|_{E \neq 0} = 2T_{\text{eff}} \left. \frac{\text{Im}\, G^R(\omega)}{-\omega} \right|_{\substack{\omega \to 0, \\ E \neq 0}}$$

fluctuation dissipation

See also, [Gursoy et al.,2010]

The fluctuation-dissipation relation at NESS is characterized by the effective temperature (at least for our systems).

Thermodynamics in NESS?

$$dE \overset{??}{=} T_{\mathrm{eff}}\, dS$$

It is highly nontrivial.

Hawking radiation (Hawking temperature) is more general than the thermodynamics of black hole.

Hawking radiation:

It occurs as far as the "Klein-Gordon equation" of fluctuation has the same form as that in the black hole.

Thermodynamics of black hole:

We need the Einstein's equation. It relies on the theory of gravity.

Summary

At least for some examples of NESS:

- There exists two temperatures in the non-linear regime.
- The effective temperature appears in terms of the Hawking temperature at the effective horizon.
- It agrees with the coefficient in the generalized fluctuation-dissipation relation in NESS.
- $T_{eff} < T$ can happen for some cases.

Time to talk beyond the research fields.

Possible directions with numerical work

The game is how to solve the non-linear partial differential equations in the gravity side.

Numerical work is definitely important.

- Numerical relativity
- But for condensed matter physics

Physics in this century

Photoelectric effect

Einstein 1905

Brownian motion

Relativity

They seemed to be independent stories.

String theory

AdS/CFT correspondence (Maldacena, 1997)

Let us take the wisdom of gravity into physics of other research field.

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